

# ATV depth estimation

Mechanical Engineering

주기영

# Project overview



Detection of drivable regions in off-road conditions

# Project overview

## Depth estimation

- Segment image into regions or objects
- Segment image into drivable/undrivable region



- Undrivable Region
- Drivable Region

# Project overview

## Depth estimation

- Calculate distance to the target region
- Designate steep incline regions or wall regions as reject region



- Undrivable Region
- Reject Region
- Drivable Region

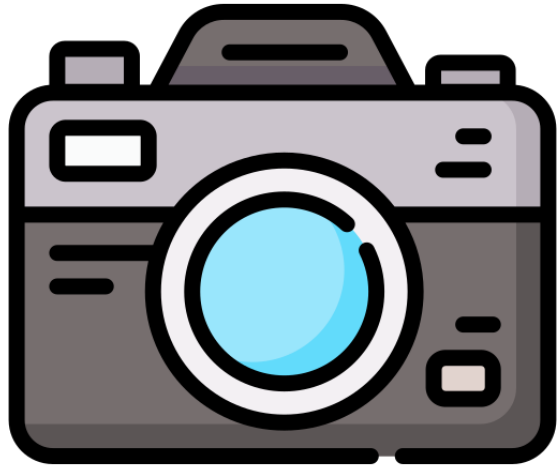
# Project overview

## For autonomous driving

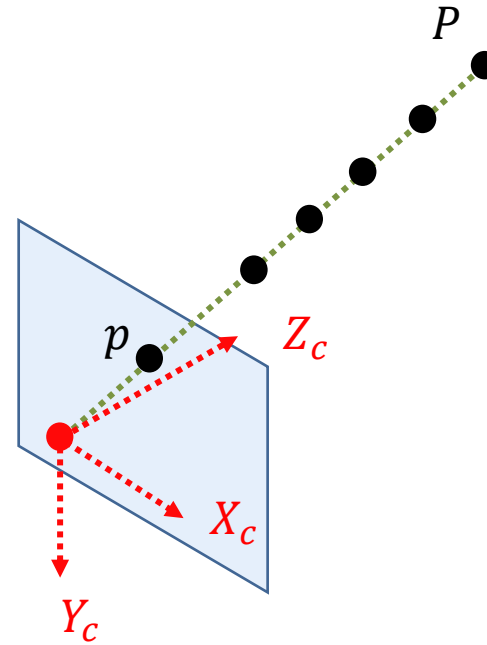
- Image real-time processing is needed

Semantic Segmentation	<ul style="list-style-type: none"><li>- Precision is more important</li><li>- Using deep learning</li></ul>
Depth estimation	<ul style="list-style-type: none"><li>- Not using deep learning for real time processing</li><li>- Stereo depth estimation</li></ul>

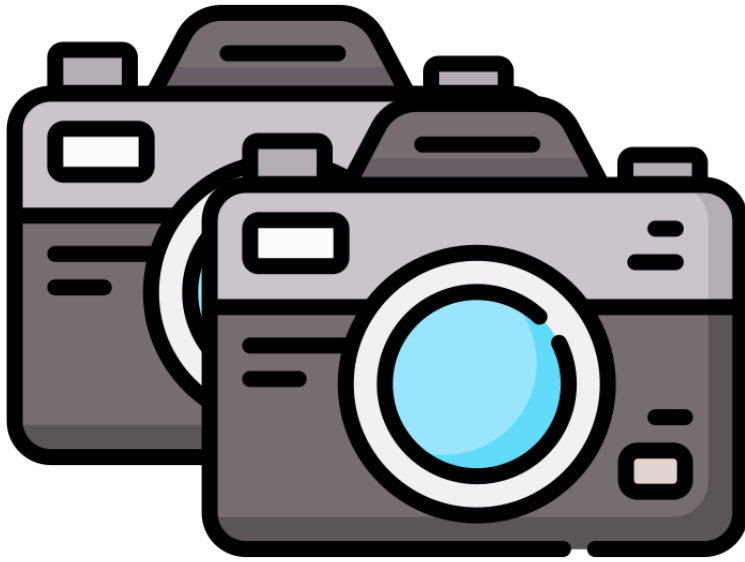
# Why do we use stereo camera?



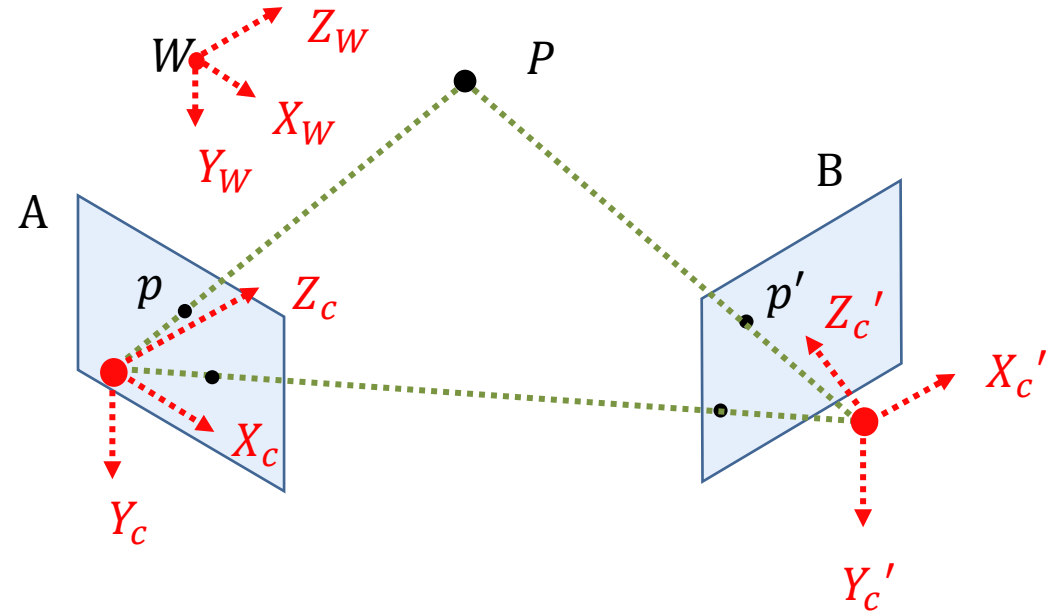
Ray of possible  
position



# Why do we use stereo camera?



**specific 3D  
coordinate**

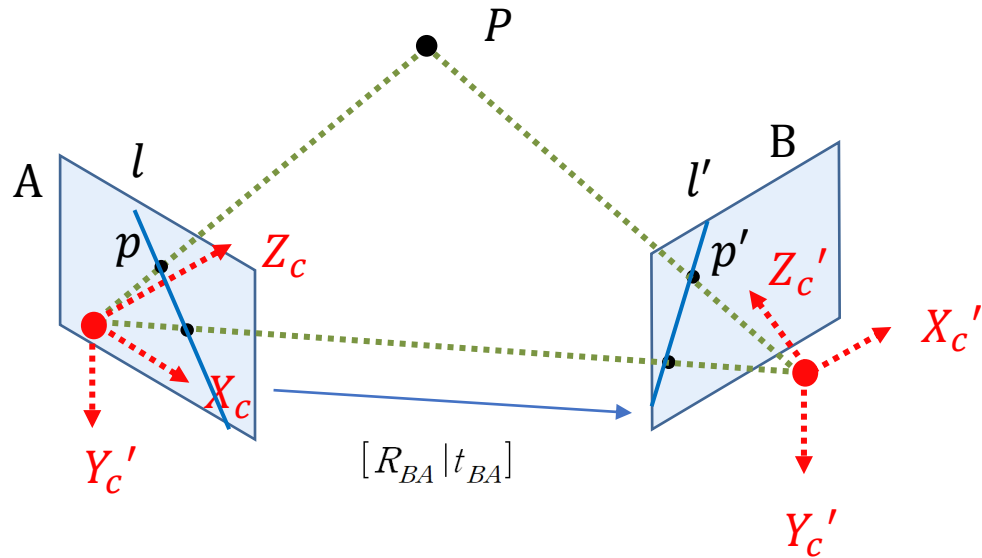


$P$  : World coordinate

$p$  : A image coordinate of P

$p'$  : A image coordinate of P

# For calculating depth



$[R_{BA} | t_{BA}]$  : Transformation matrix

→ **Camera calibration**

$p$  : A image coordinate of  $P$

$p'$  : A image coordinate of  $P$

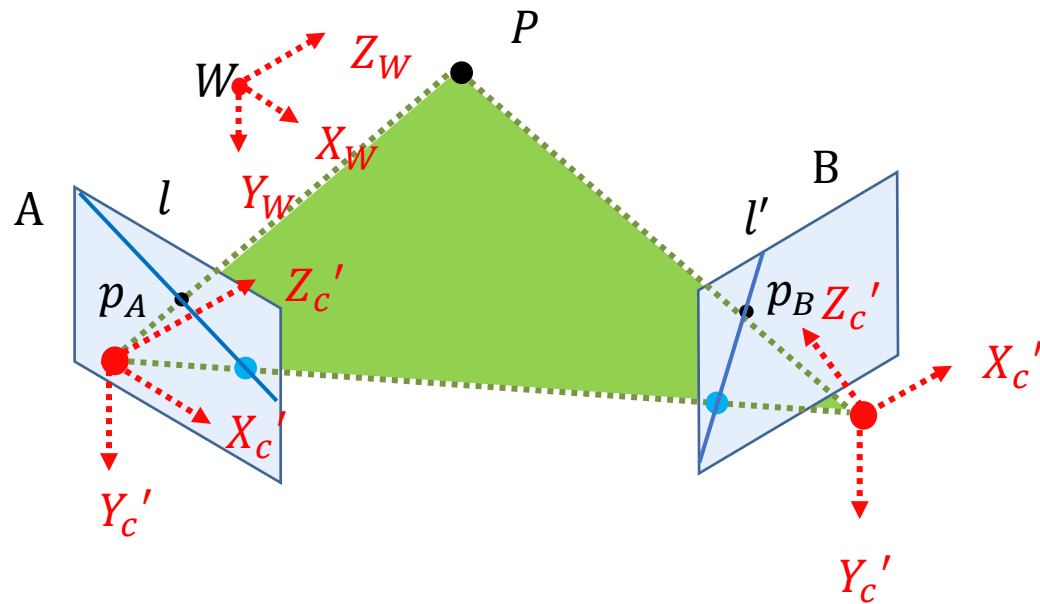
→ **Epipolar geometry**

→ **We can find world coordinate of  $P$**



# Epipolar Geometry

The geometric relationship between two camera views of the same 3D Point

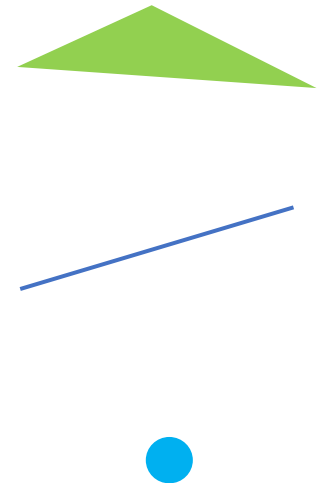


- $P$  : World coordinate
- $p_A$  : Point projected onto camera A
- $p_B$  : Point projected onto camera B

**Epipolar plane**

**Epiline (Epipolar line)**

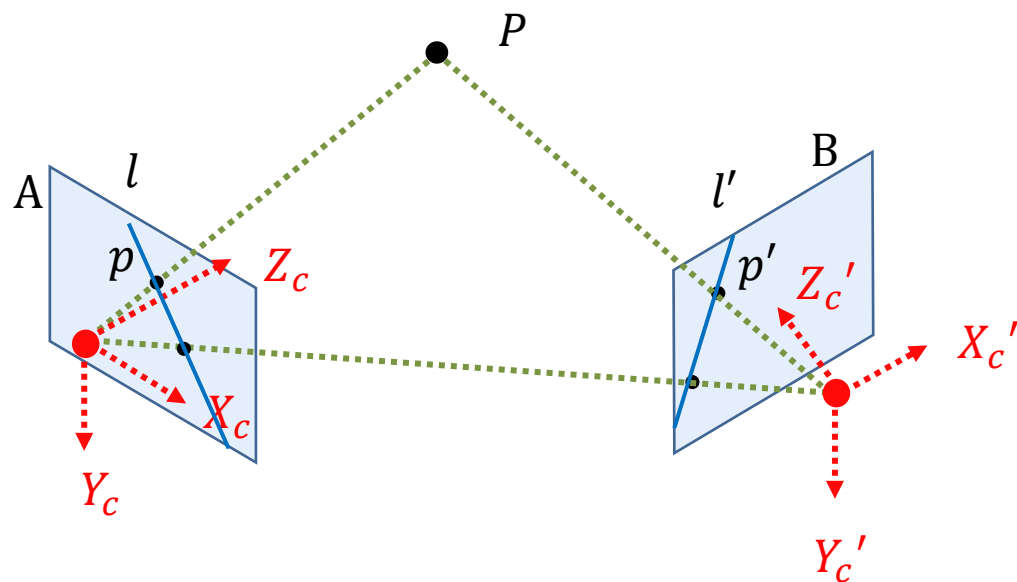
**Epipole**



# Epipolar Geometry

## Correspondence point

Pairs of image points representing the same world point



Correspondence point

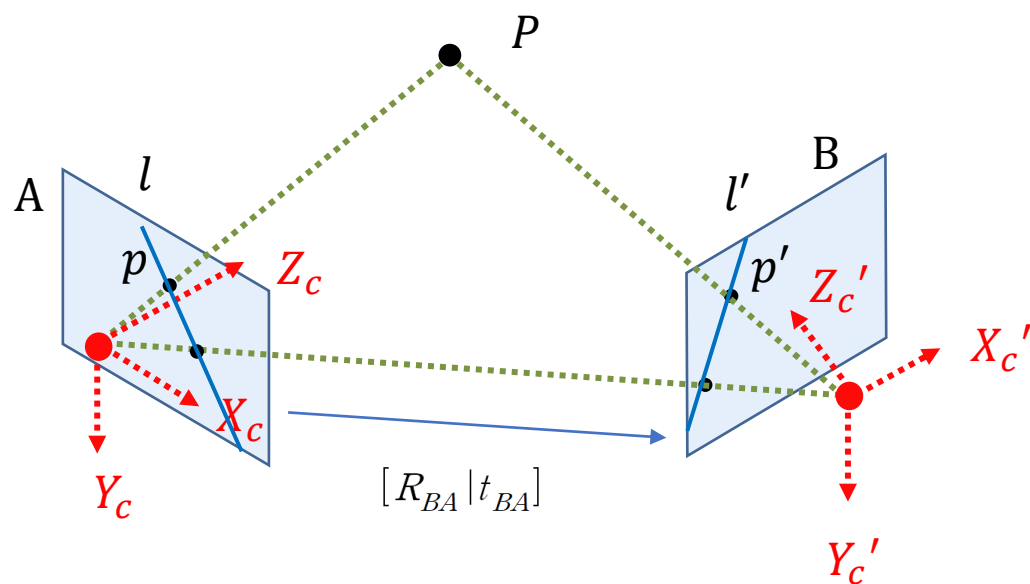
$p$   $\longleftrightarrow$   $p$

$p$  lies on the epiline  $l$   
 $p'$  lies on the epiline  $l'$

# Epipolar Geometry

## Essential Matrix

matrix that relates corresponding points between two images



$$[t]_x = \begin{pmatrix} 0 & -t_1 & t_2 \\ t_1 & 0 & -t_3 \\ -t_2 & t_3 & 0 \end{pmatrix}$$

$p$  : A image coordinate of P

$p'$  : A image coordinate of P

$E$  : Essential matrix

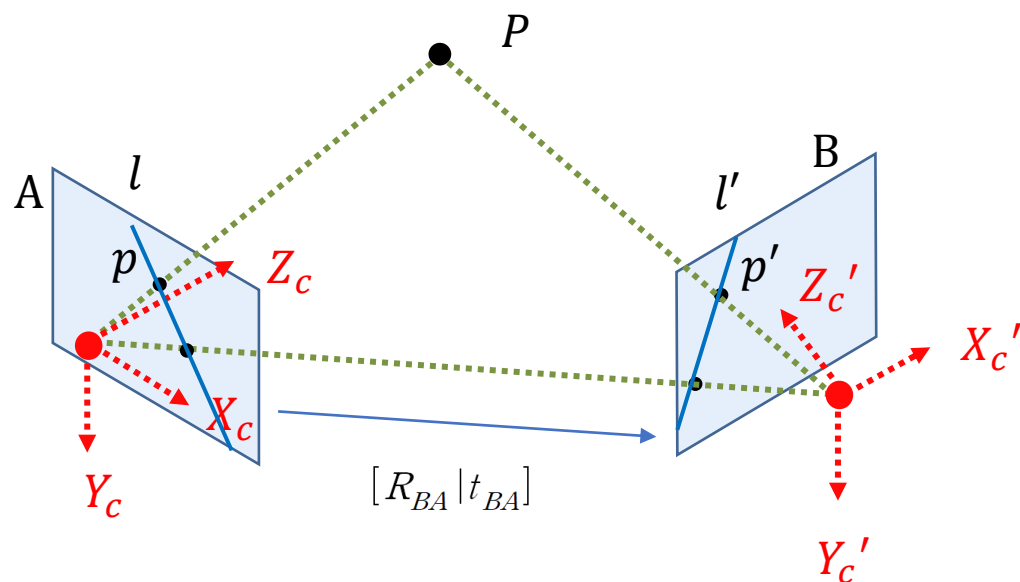
$[R_{BA}|t_{BA}]$  : Transformation matrix

$$E = [t_{BA}]_x R_{BA} \longrightarrow p'^T E p = 0$$

# Epipolar Geometry

## Projective geometry

$u$ : A line	$\longrightarrow$	$x^T u = 0$
$x$ : A point on a line $u$		



By the definition of epiline  $\longrightarrow p'^T l' = 0$

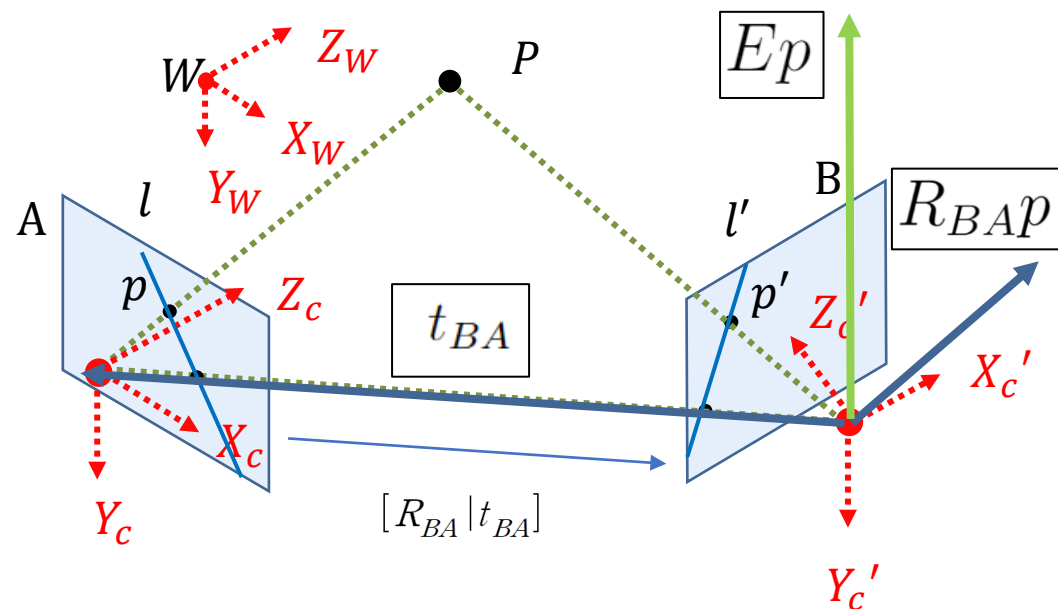
$$p'^T E p = 0$$

$$\begin{cases} p' & : \text{Correspondence point} \\ E p & : \text{Epiline } l' \end{cases}$$

# Epipolar Geometry

Meaning of  $Ep$  vector

$$E = [t_{BA}]_x R_{BA} \longrightarrow Ep = [t_{BA}]_x R_{BA} p$$



**3D World Coordinate system**

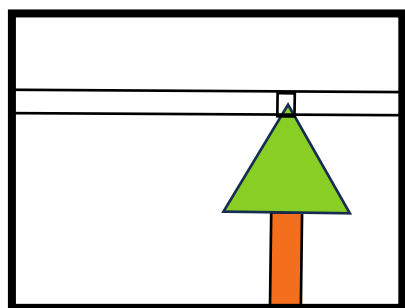
Normal vector of the Epipolar plane

**B Image coordinate system**

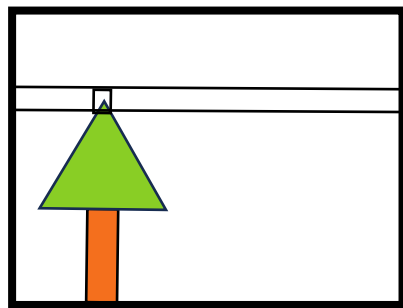
Homogeneous expression of Epiline

# Depth Estimation

## Finding correspondence

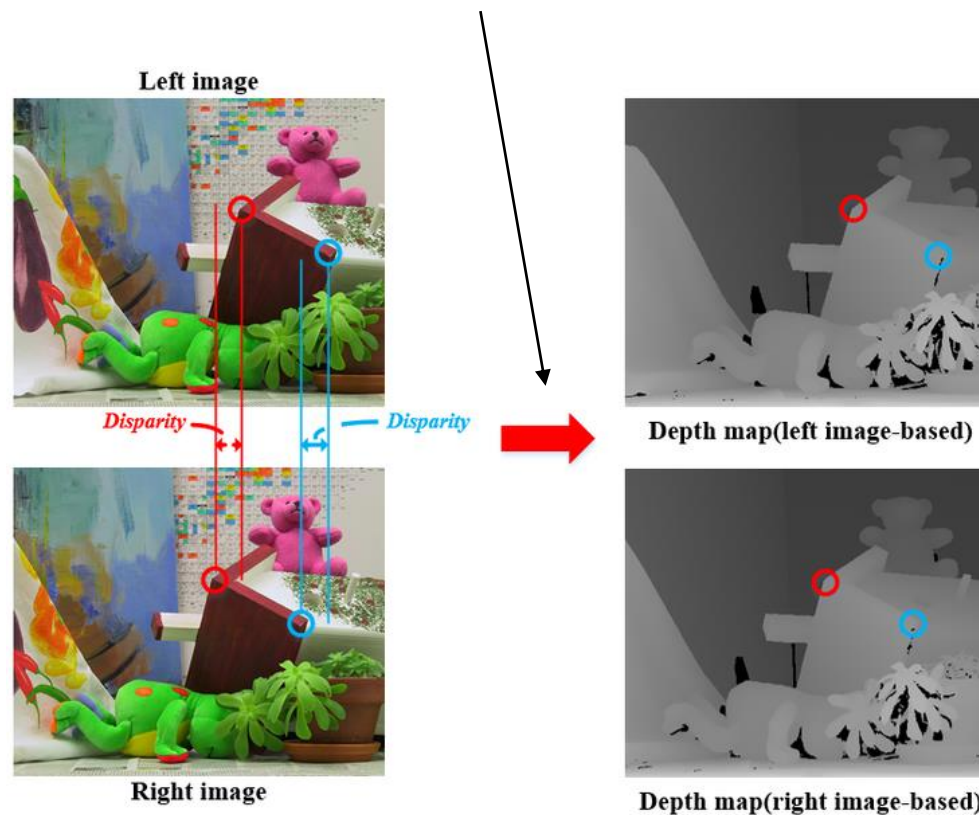


Reference  
Image



Target  
Image

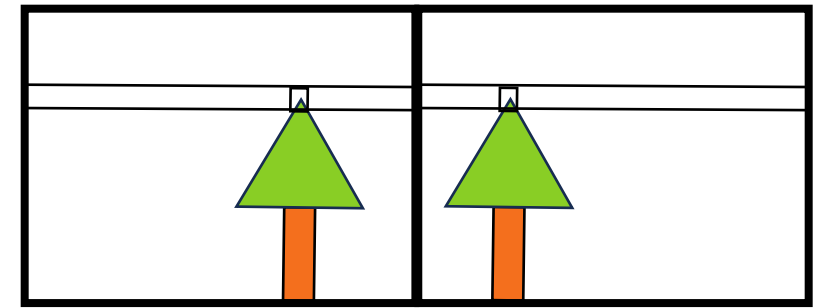
## Triangulization



# Depth Estimation

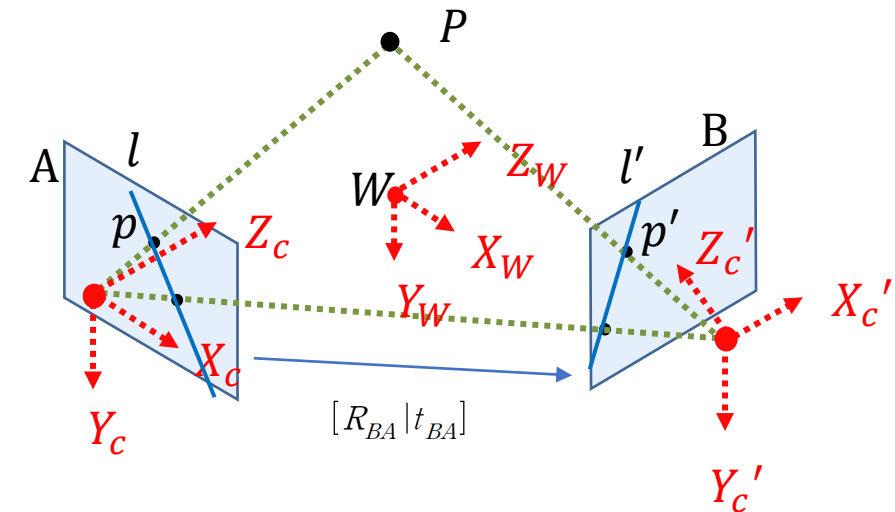
## Step 1 : Correspondence Matching

- The process of calculating  $p'$



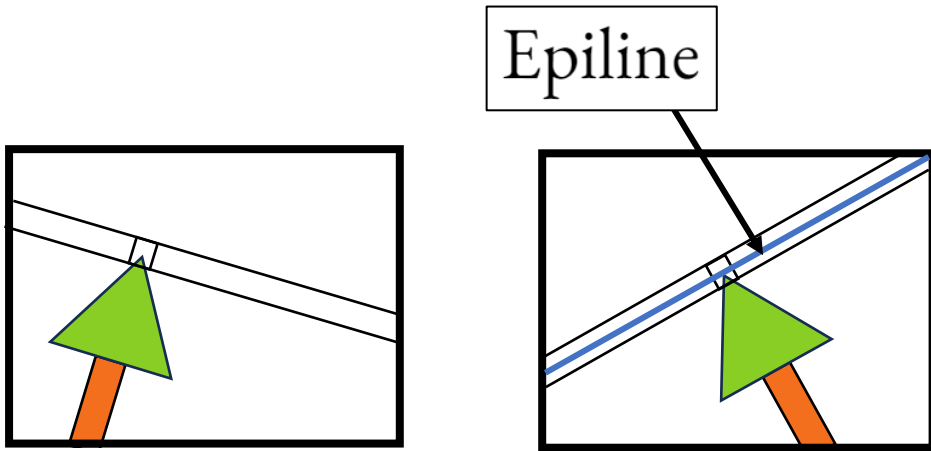
## Step 2 : Triangulization

- The process of calculating world coordinate of  $P$



# Correspondence Matching

## Using Epipolar geometry



### Finding Epiline

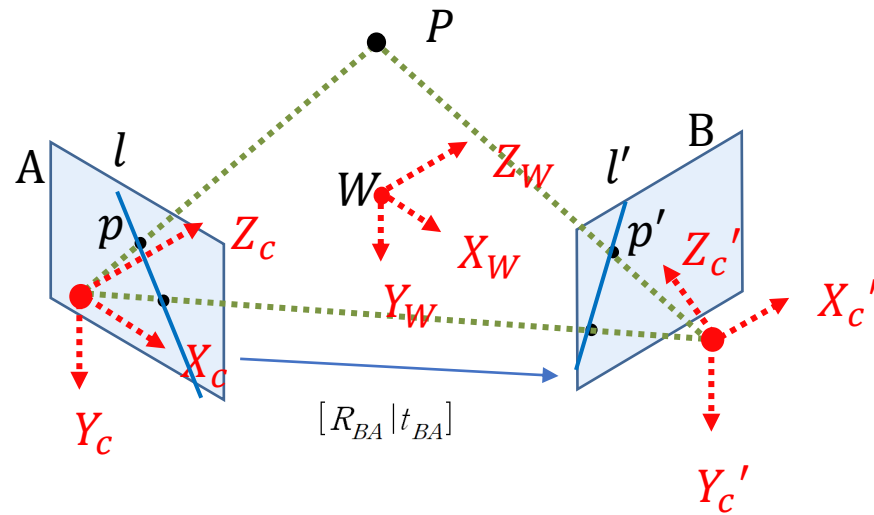
Using Essential matrix

### Finding correspondence point

Using Block matching



# Triangulation



## Known

### Camera calibration

$[R_W A | t_W A]$  : Extrinsic parameter of camera A

$[R_W B | t_W B]$  : Extrinsic parameter of camera B

### Correspondence matching

$p$  : A image coordinate of P

$p'$  : A image coordinate of P

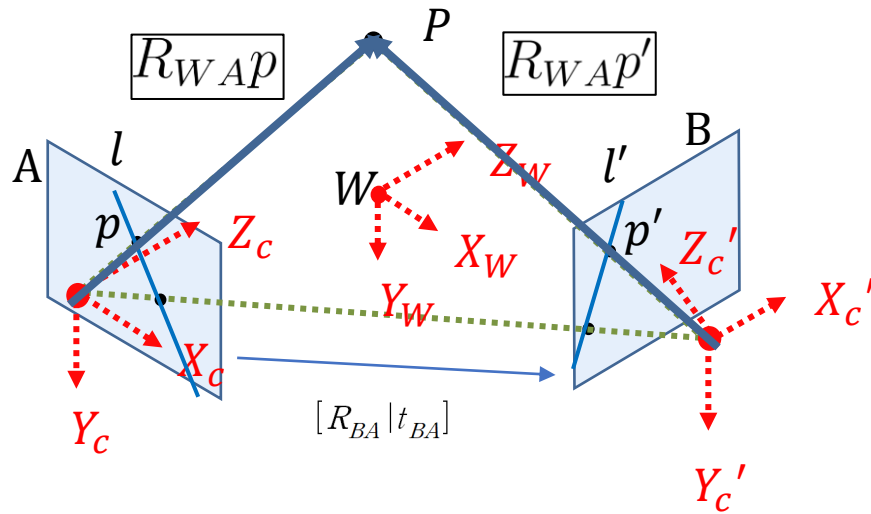
## Unknown

$P$  : World coordinate

The process of finding world coordinate

# Triangulation

## Line expression



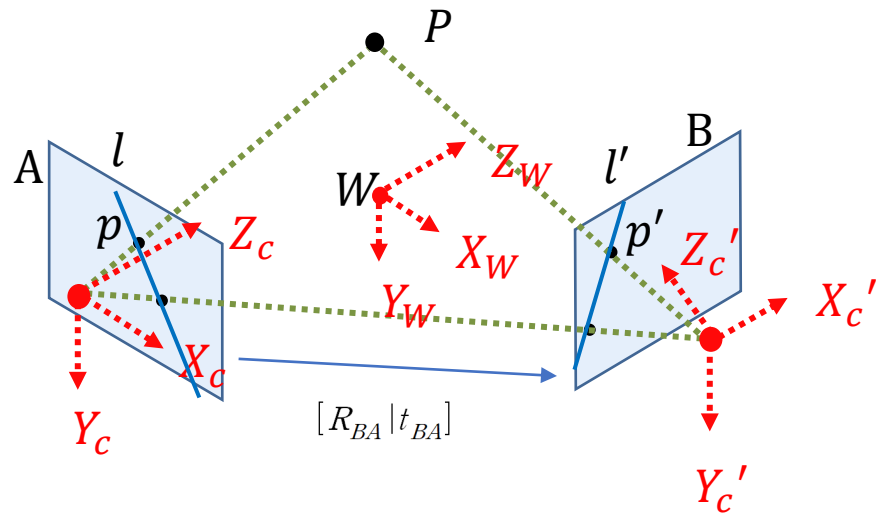
$$m : su + p$$

$$\begin{cases} m & : \text{Line} \\ u & : \text{Direction vector} \\ p & : \text{point} \end{cases}$$

$$m_A : s_A R_{WAp} + t_{WA}$$

$$m_B : s_B R_{WAp'} + t_{WA}$$

Intersection point of two lines,  $m_1$  and  $m_2$   $\longrightarrow$   $P$



Directly using two image with  
arbitrary camera position

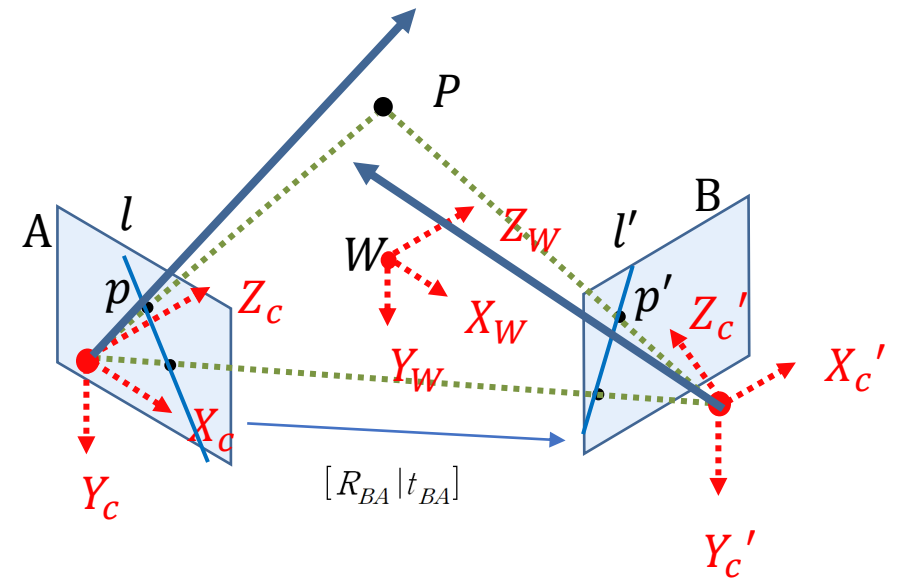
→ Some **considerations**

# Considering

## 1. Existence of intersection point

- The loss of spatial information continuity
- Camera calibration Error, Pixel Noise

→ The intersection of the two lines does not exist

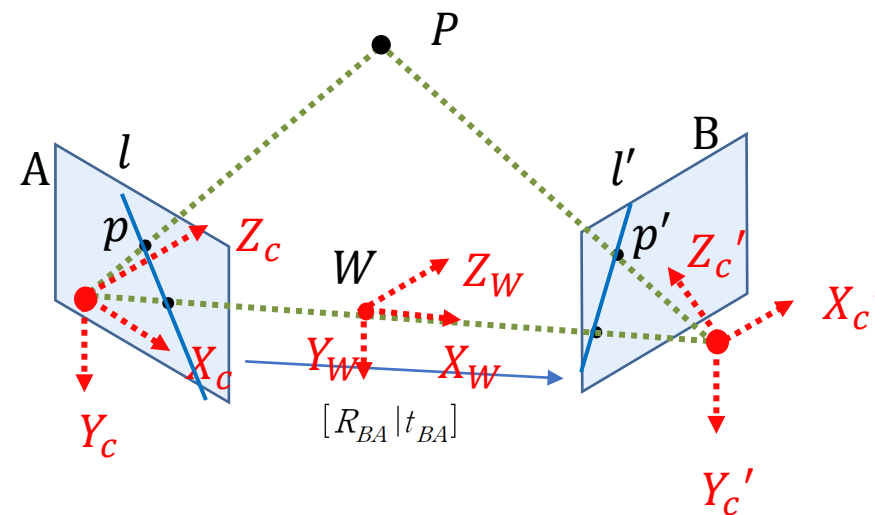


# Considering

## 2. We can set 1D coordinate

- we don't know all world coordinate
- We can set proper coordinate for simple problem

→ For calculating depth, we only know one coordinate of  $z$



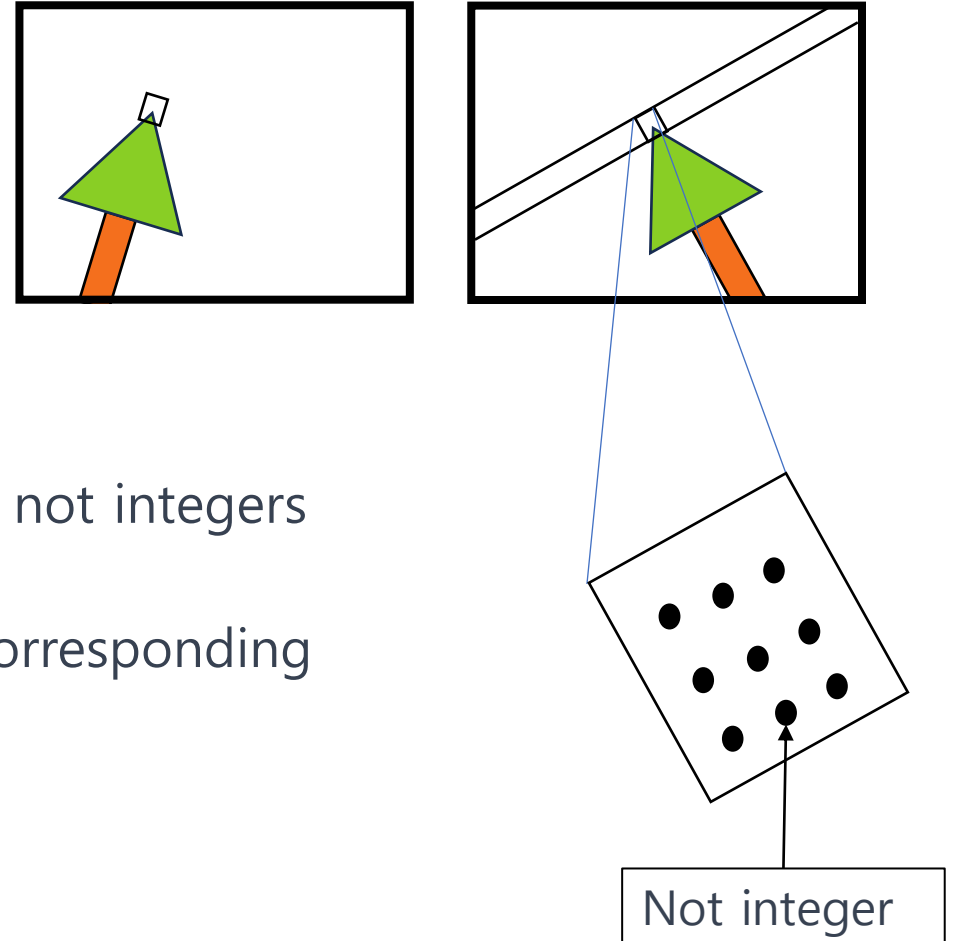
$X_W$  : Baseline direction

$Z_W$  : Desired Depth direction

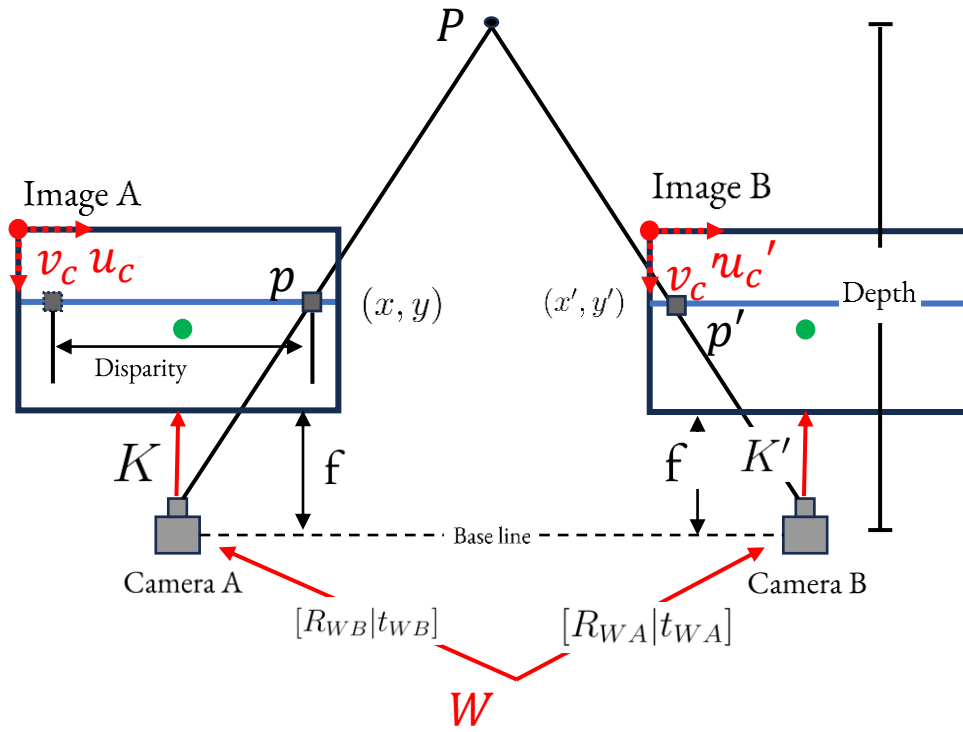
# Considering

## 3. Problem with Block matching Algorithm

- The coordinates of the points inside the block are not integers
- Epiline is formed diagonally, it is difficult to find corresponding points along the Epiline



# Triangulization(Ideal Modeling)



## Camera calibration

$$\lambda \tilde{m} = K [R_{CW} | t_{CW}] \tilde{w} \quad \left\{ \begin{array}{l} \tilde{m} : \text{Image coordinate} \\ K : \text{Intrinsic parameter} \\ [R_{CW} | t_{CW}] : \text{Extrinsic parameter} \\ \tilde{w} : \text{World coordinate} \end{array} \right.$$

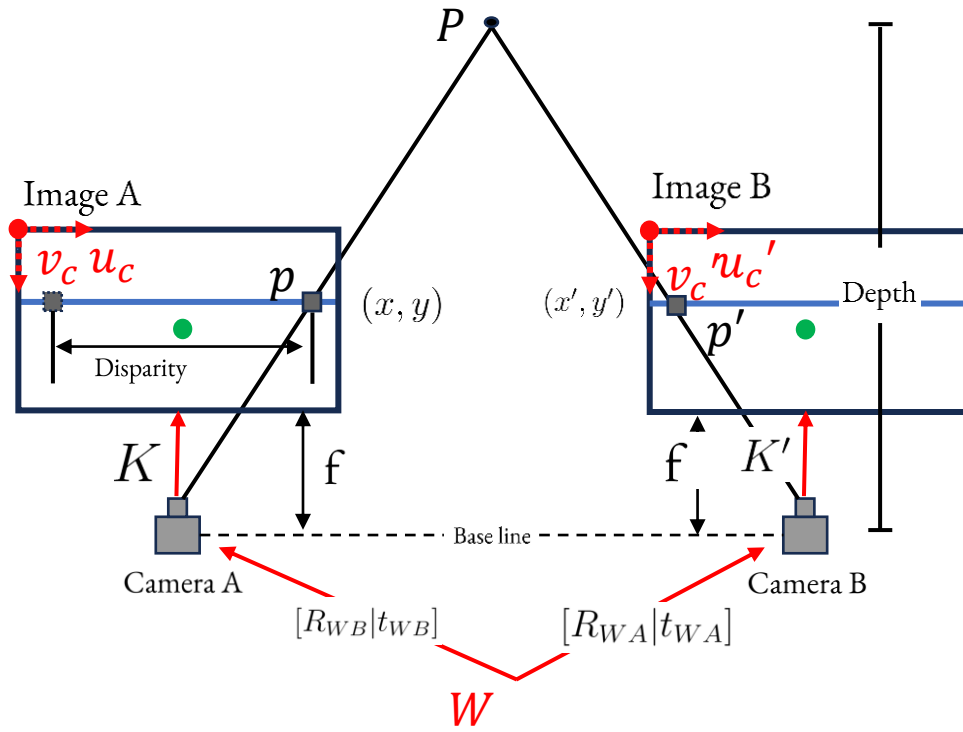
## Same Intrinsic parameter

$$K = K' \longrightarrow \left\{ \begin{array}{l} \text{Same principal point coordinate} \\ \text{Same Image ratio} \end{array} \right.$$

## Same Rotation matrix

$$R_{WA} = R_{WB} \longrightarrow \text{Same camera posture}$$

# Triangulation(Ideal Modeling)



## Important Effect of Ideal Modeling

- The two image planes exist within the same plane
- All Epiline is always parallel to the horizontal axis

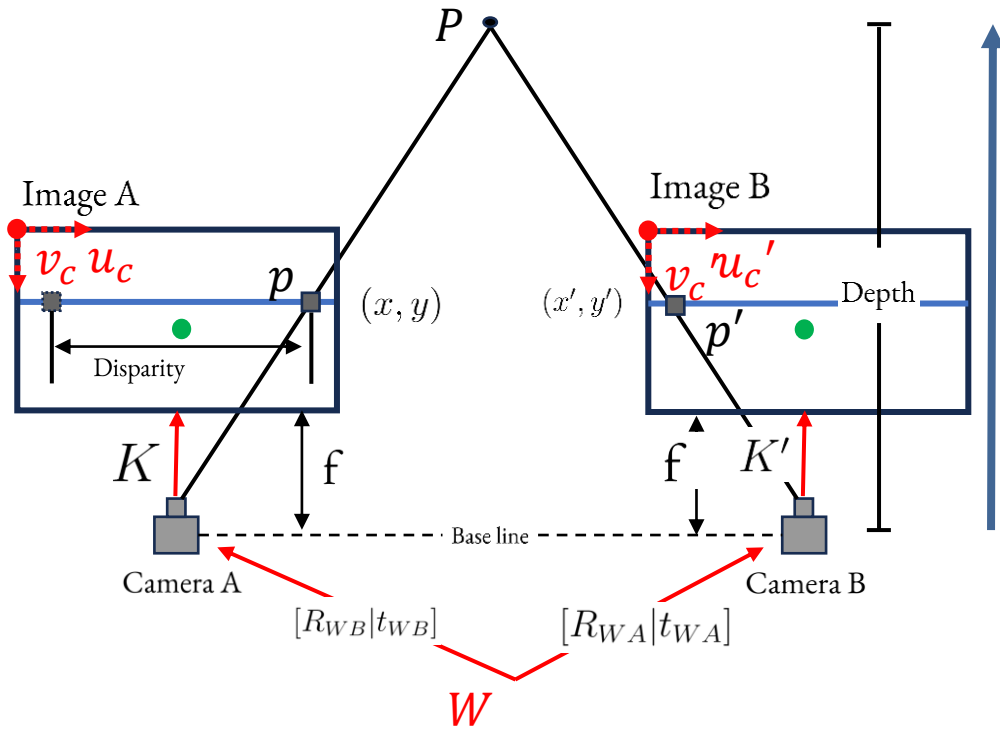
## Because

Intersection line of Epipolar plane and image plane are always parallel to Baseline



# Triangulation(Ideal Modeling)

Recall three considerations



## Existence of intersection point

We don't consider intersection point

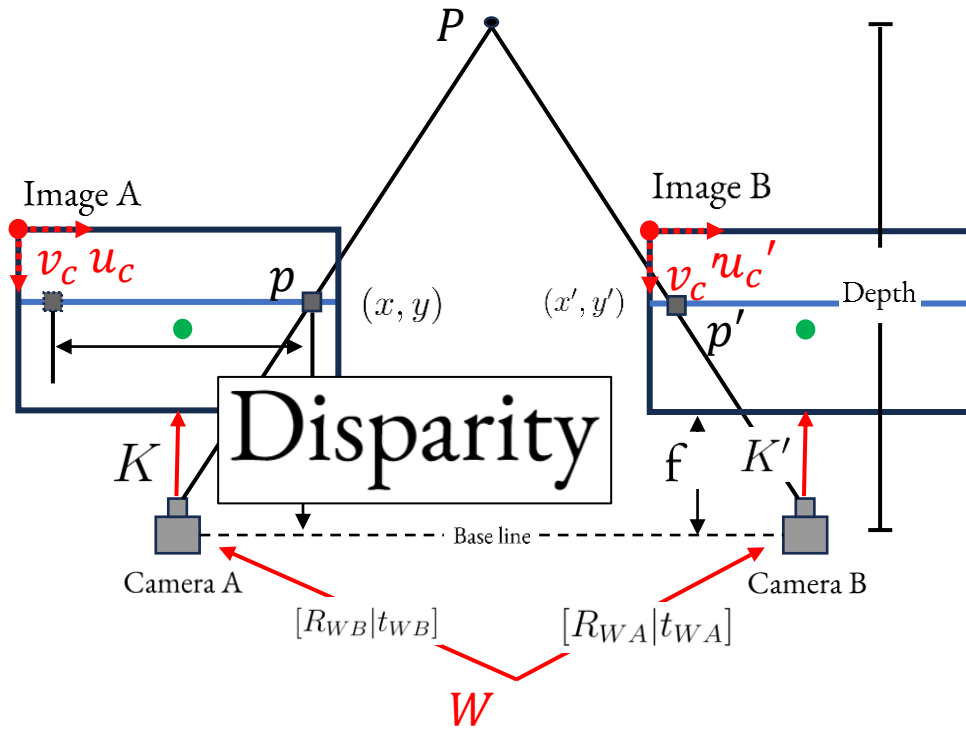
## We can set 1D coordinate

Depth calculation in the direction the camera is facing is very easy

## Problem with Block matching Algorithm

Epiline is parallel to the horizontal axis

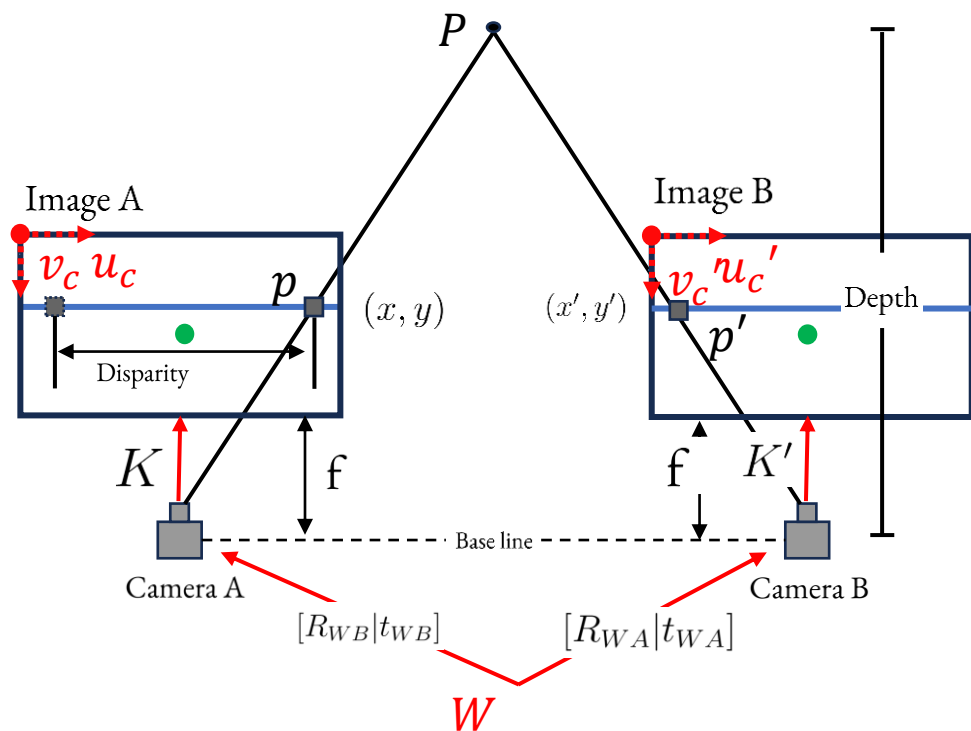
# Triangulation(Ideal Modeling)



## Disparity

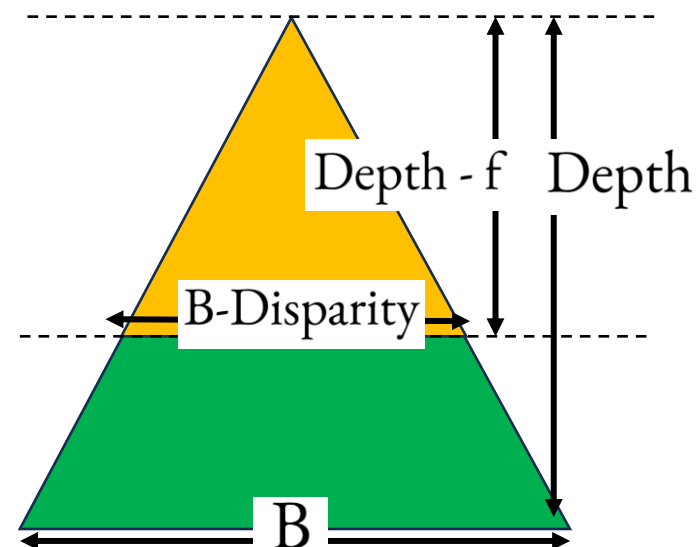
horizontal pixel shift between correspondence point in a pair of stereo images

# Triangulation(Ideal Modeling)



$$Depth - f : Depth = B - Disparity : B$$

## Simple Triangulation

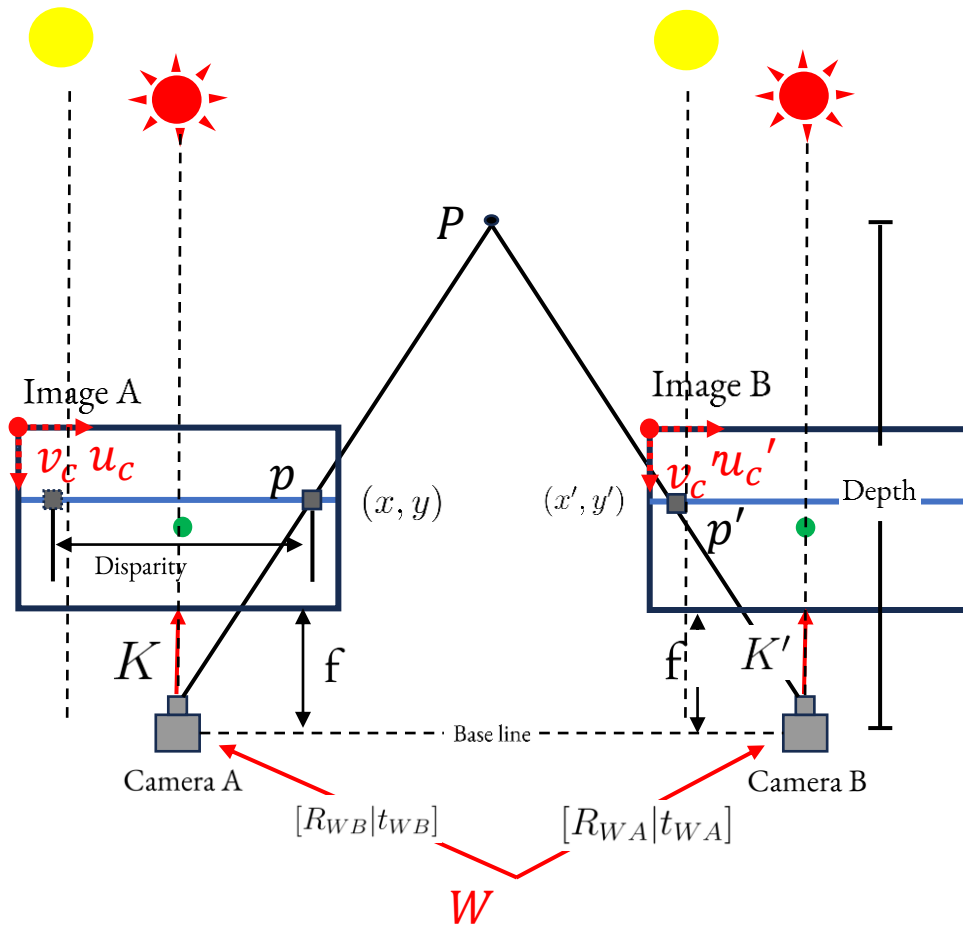


$$\longrightarrow Depth = \frac{f \times B}{Disparity}$$

In ideal modeling

Why should the Intrinsic parameters  
be the same?

# Triangulation(Ideal Modeling)

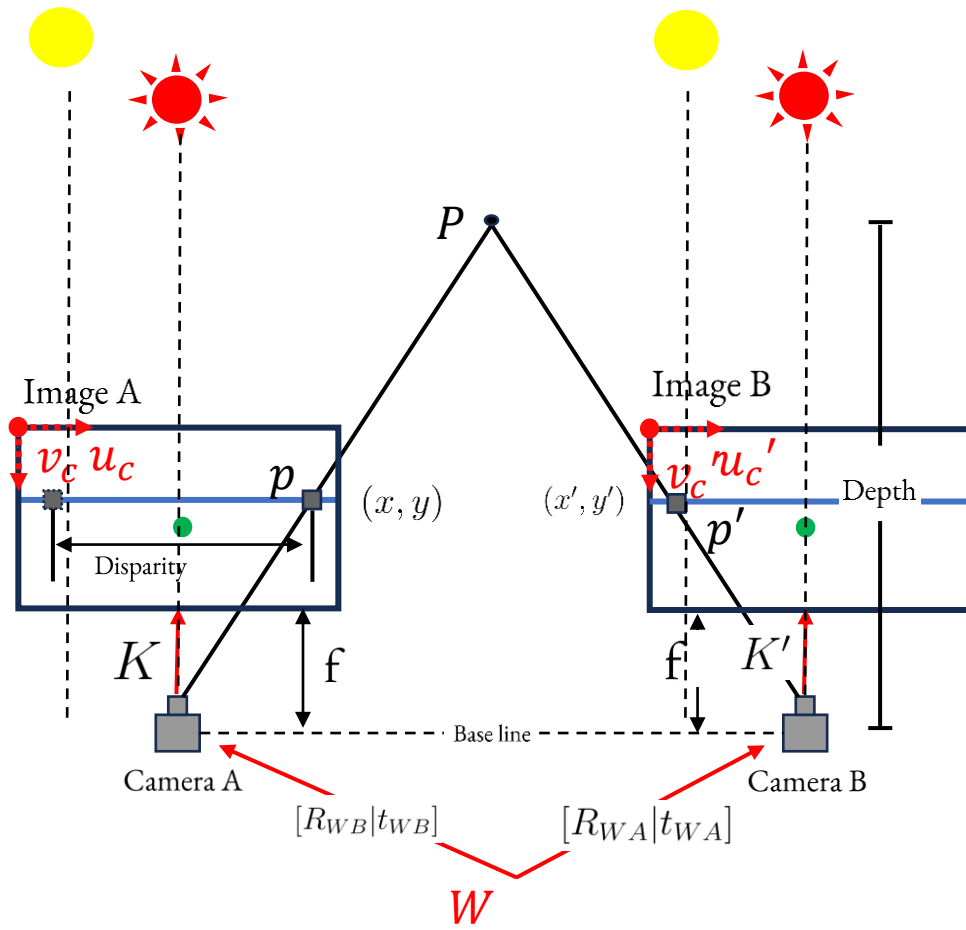


$$Depth = \frac{f \times B}{Disparity}$$

$$\lim_{Depth \rightarrow \infty} Disparity = 0$$

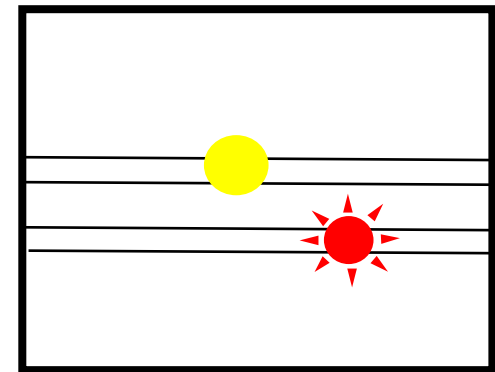
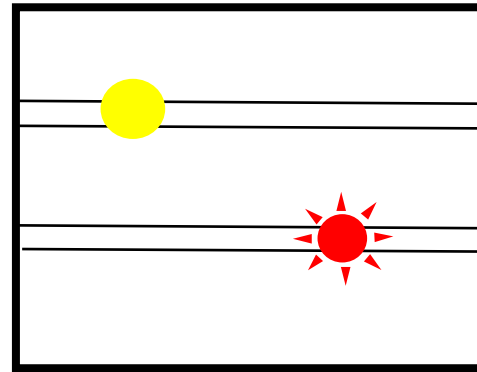
→ Disparity of objects that have very high depth is 0

# Triangulation(Ideal Modeling)



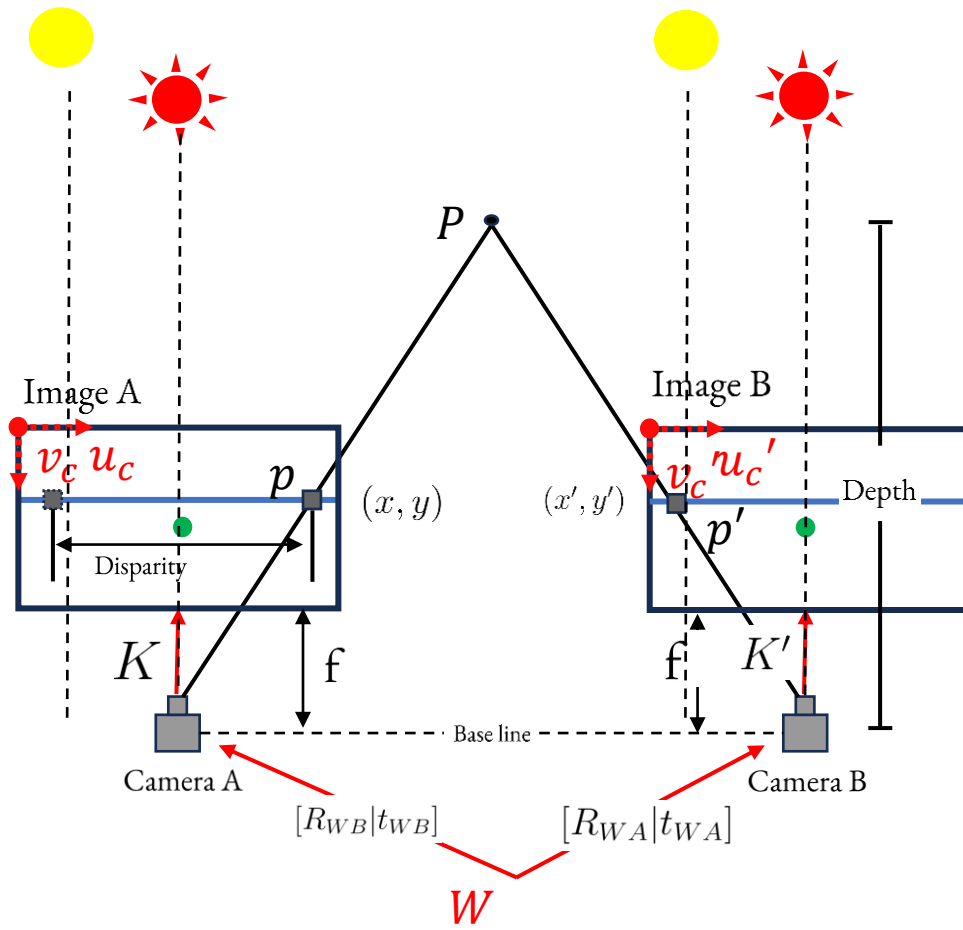
**Case 1 (Different  $f_x, f_y$ )**

$$K = \begin{pmatrix} f_x & 0 & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{pmatrix}$$



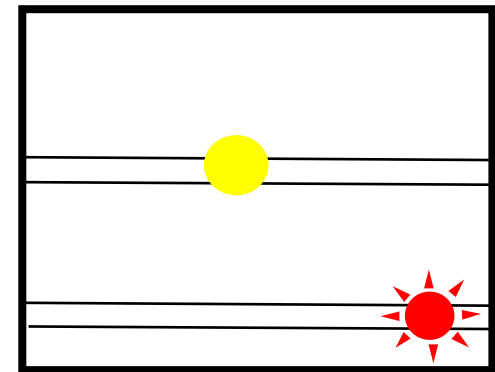
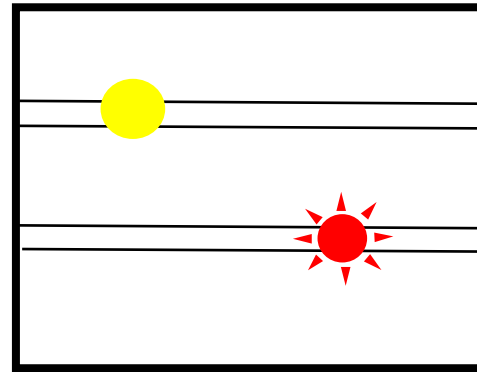
The Epipolar lines of the moon do not have the same vertical axis coordinate

# Triangulation(Ideal Modeling)



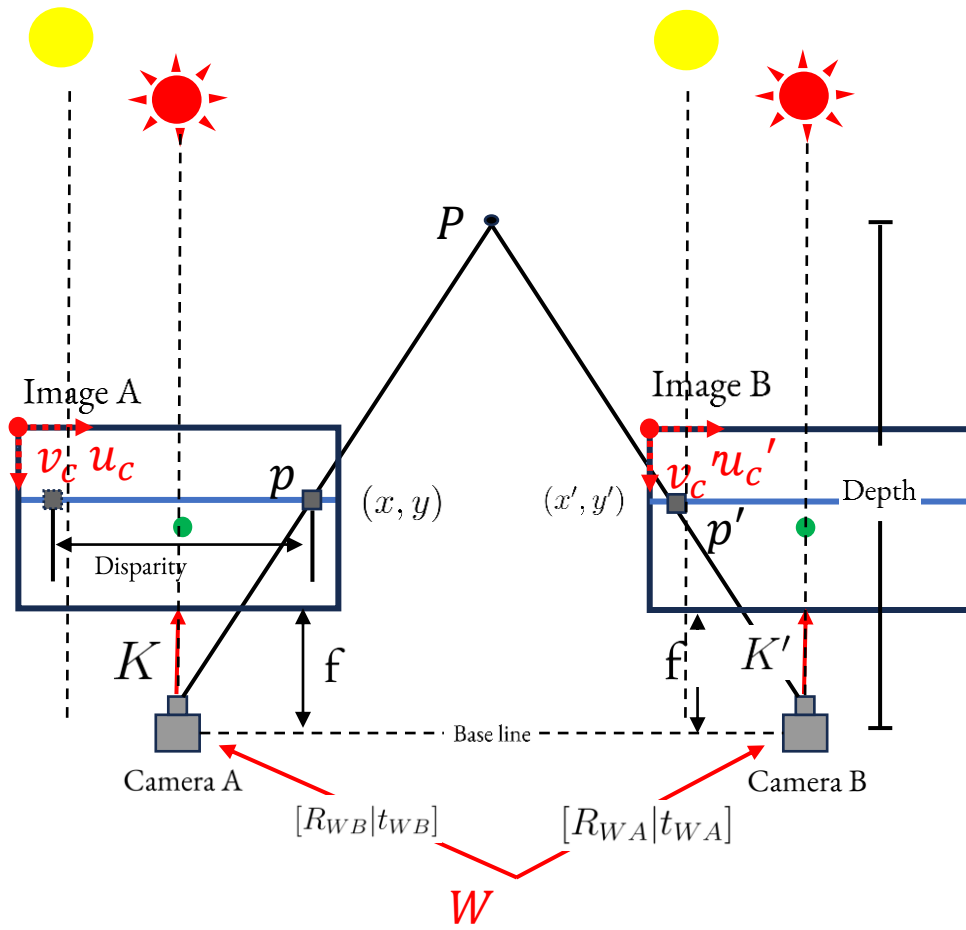
Case 2 (Different  $c_x, c_y$ )

$$K = \begin{pmatrix} f_x & 0 & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{pmatrix}$$

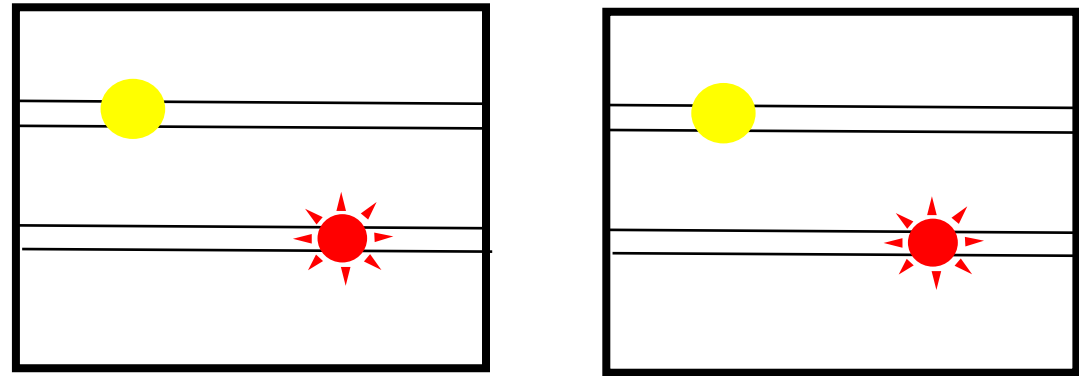


The Epipolar lines of both sun and moon do not have the same vertical axis coordinate

# Triangulization(Ideal Modeling)



## Case 3 (Same Intrinsic parameter)





# Triangulization(Ideal Modeling)

## Conclusion

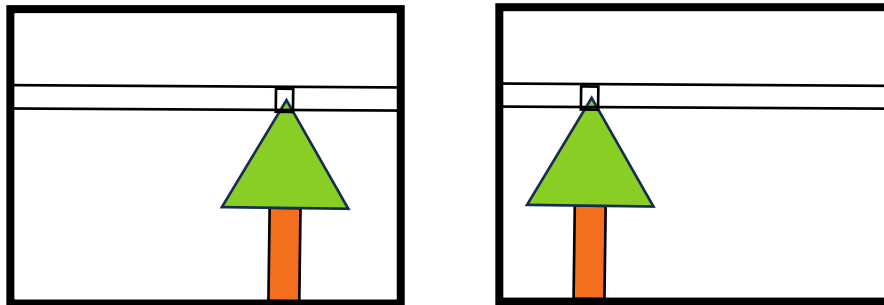
### Same Intrinsic parameter

- All Epiline is always parallel to the horizontal axis

### Same Camera

- All pair of correspondence points share the same vertical axis coordinates

### Simple correspondence matching



### Simple triangulization

$$Depth = \frac{f \times B}{Disparity}$$

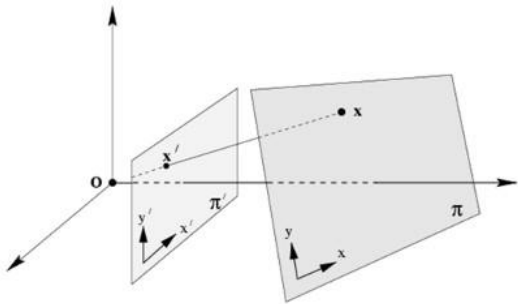
We have stereo images from  
various camera position

→ Rectification

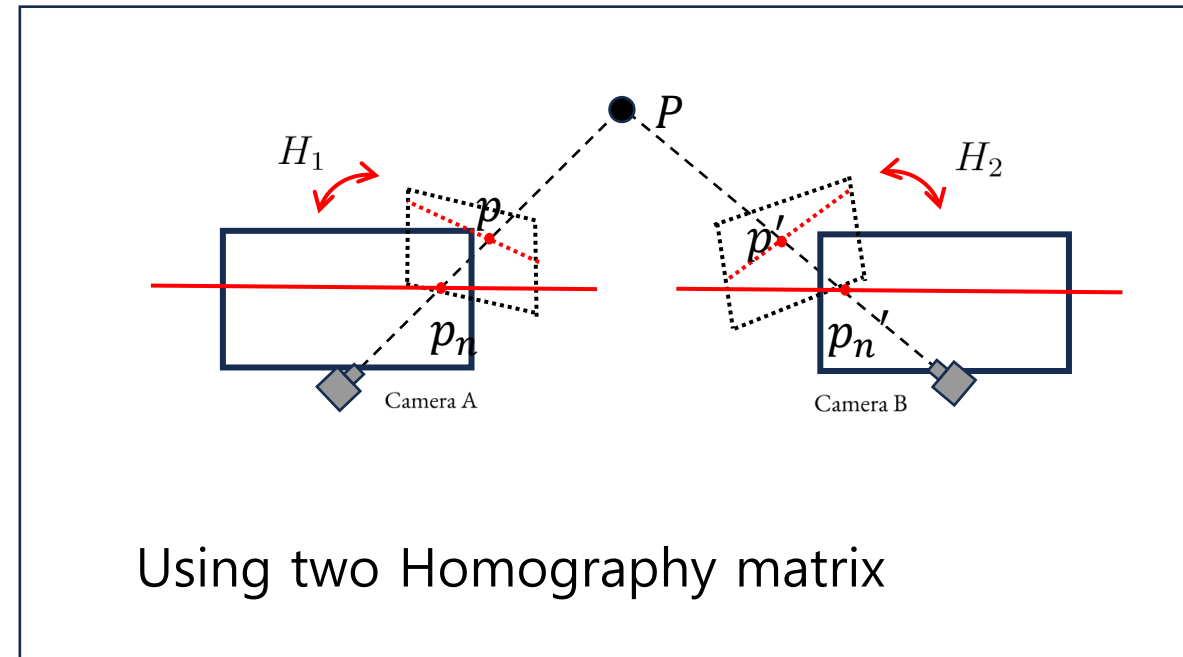
# Rectification

## Homography

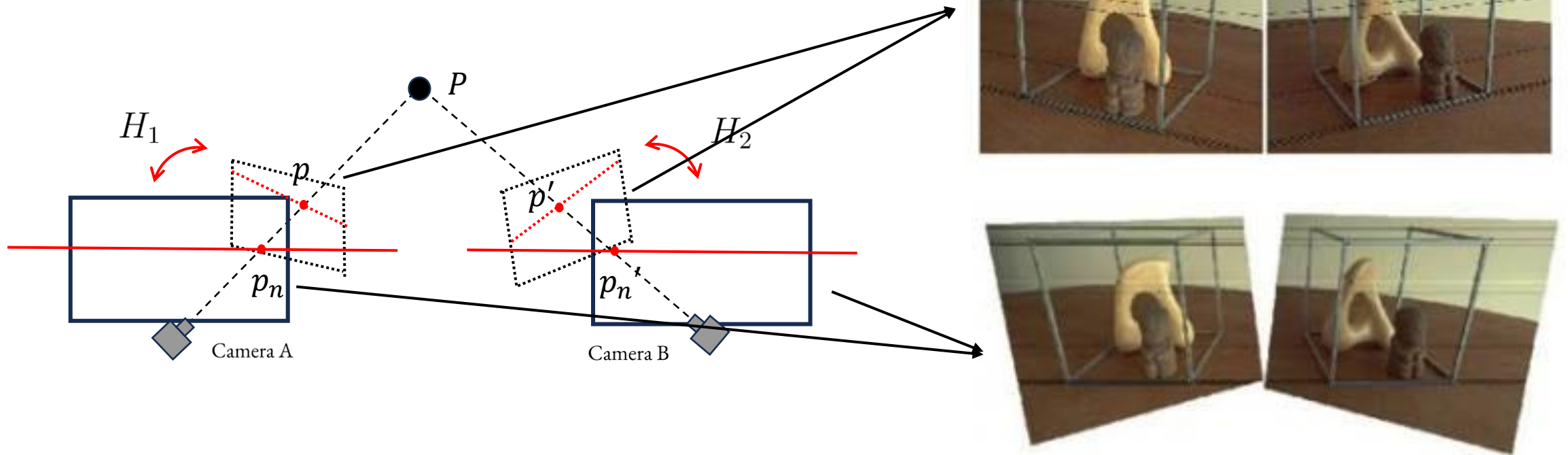
- 3x3 matrix representing a projective transformation between two planes
- Two images taken at the same location can be overlapped using homography



$$w \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$



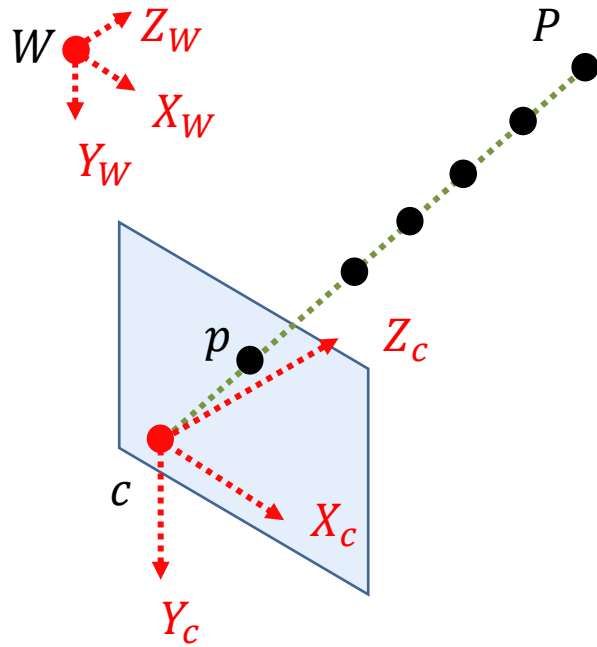
# Rectification



## Object

- All apilines are parallel to the horizontal axis of the image plane
- Two image coordinates for the same point  $P$  have the same vertical coordinates

# Rectification



## Camera calibration

$$\lambda \tilde{m} = K [R_{CW} | t_{CW}] \tilde{w} \quad \left\{ \begin{array}{l} \tilde{m} : \text{Image coordinate} \\ K : \text{Intrinsic parameter} \\ [R_{CW} | t_{CW}] : \text{Extrinsic parameter} \\ \tilde{w} : \text{World coordinate} \end{array} \right.$$

## Camera center

$$c = -R_{CW}^{-1} t_{CW} \quad c : \text{World coordinate of camera center}$$

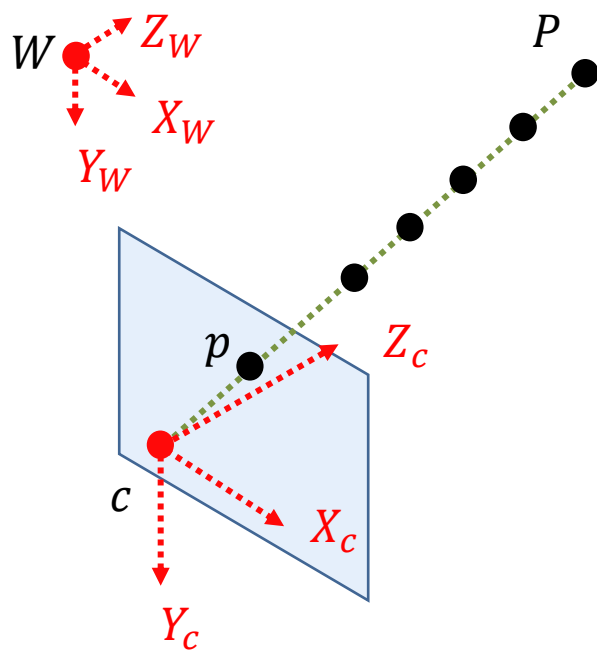
$$t_{CW} = -R_{CW} c$$

## Projection matrix

$$\tilde{P} = K [R_{CW} | t_{CW}] = K [R_{CW} | -R_{CW} c] \quad (\text{where } Q = K R_{CW})$$

$$\tilde{P} = [Q | -Qc]$$

# Rectification



## Camera calibration

$$\lambda \tilde{m} = K [R_{CW} | -R_{CW}c] \tilde{w} \quad \left\{ \begin{array}{l} \tilde{m} : \text{Image coordinate} \\ K : \text{Intrinsic parameter} \\ [R_{CW} | t_{CW}] : \text{Extrinsic parameter} \\ \tilde{w} : \text{World coordinate} \end{array} \right.$$

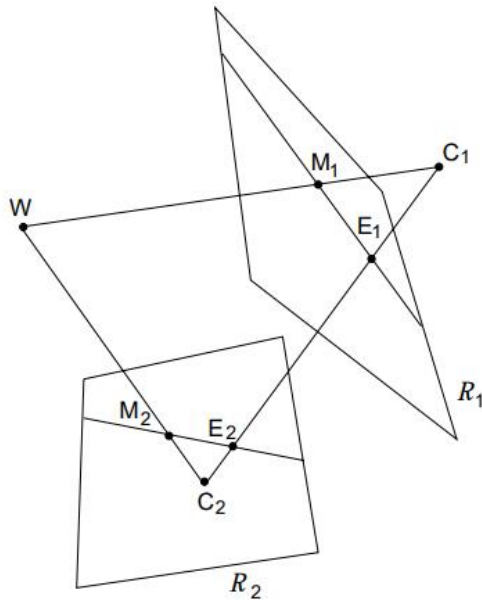
## World coordinate

$$\tilde{w} = \begin{pmatrix} w \\ 1 \end{pmatrix} \longrightarrow \lambda \tilde{m} = K [R_{CW} | -R_{CW}c] \begin{pmatrix} w \\ 1 \end{pmatrix}$$

$$\lambda \tilde{m} = KR_{CW} (w - c)$$

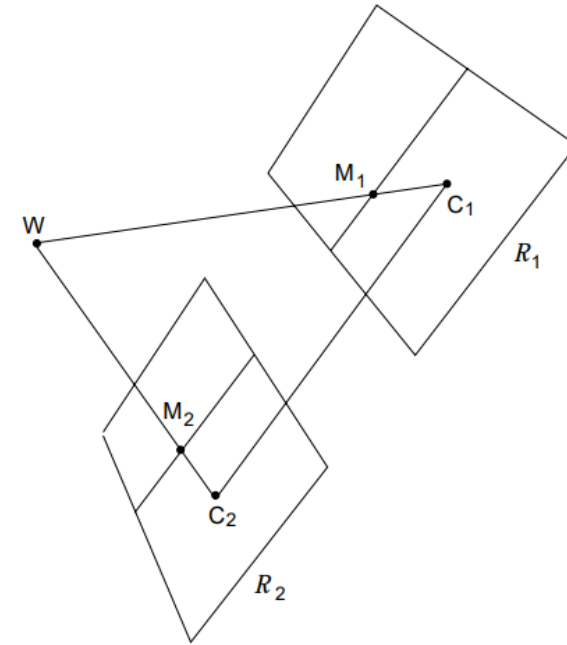
$$w = c + (KR_{CW})^{-1} \lambda \tilde{m}$$

# Rectification



$$\tilde{P}_o1 = K_{o1}[R_{o1} | -R_{o1}c_1]$$

$$\tilde{P}_o2 = K_{o2}[R_{o2} | -R_{o2}c_2]$$

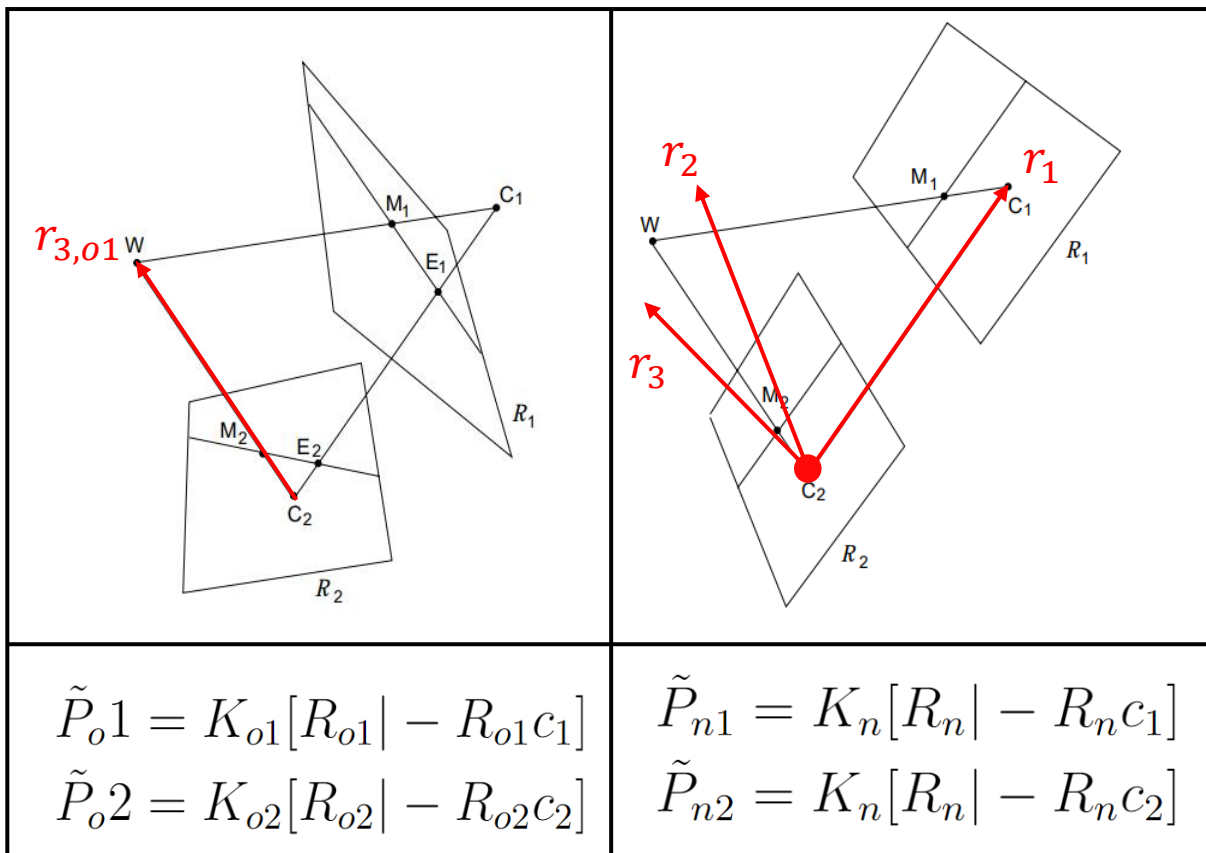


$$\tilde{P}_{n1} = K_n[R_n | -R_n c_1]$$

$$\tilde{P}_{n2} = K_n[R_n | -R_n c_2]$$

Rectification : The process of making a virtual camera

# Rectification



$R_n$

$$R = \begin{bmatrix} r_1^T \\ r_2^T \\ r_3^T \end{bmatrix} \quad \left\{ \begin{array}{ll} r_1 & : \text{camera x-dir (world expression)} \\ r_2 & : \text{camera y-dir (world expression)} \\ r_3 & : \text{camera z-dir (world expression)} \end{array} \right.$$

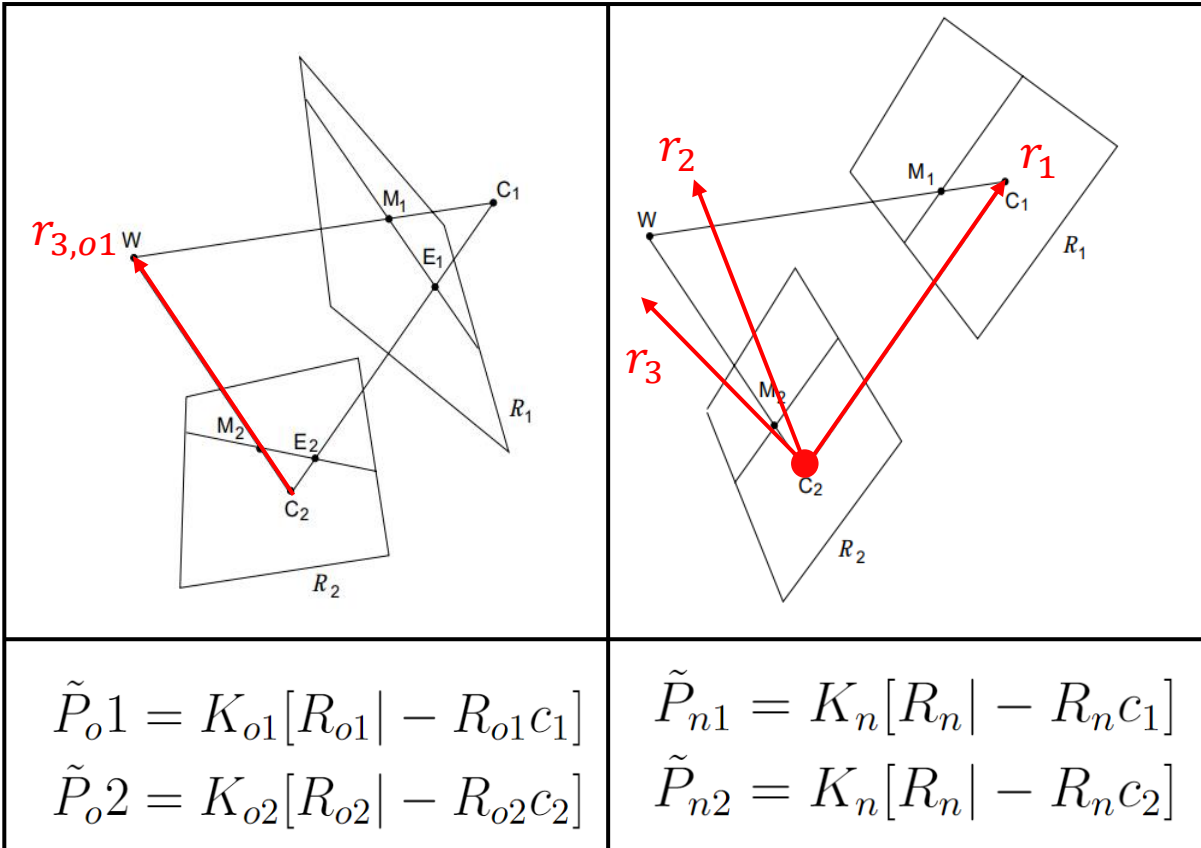
$$\begin{array}{l} \tilde{P}_o1 = [Q_{o1}| - Q_{o1}c_1] \\ \tilde{P}_o2 = [Q_{o2}| - Q_{o2}c_2] \end{array} \longrightarrow C_1, C_2$$

$$r_1 = \frac{c_1 - c_2}{\|c_1 - c_2\|} \quad r_2 = r_{3,o1} \times r_1 \quad r_3 = r_1 \times r_2$$

$r_{3,o1}$  : The principal axis dir (old camera)



# Rectification

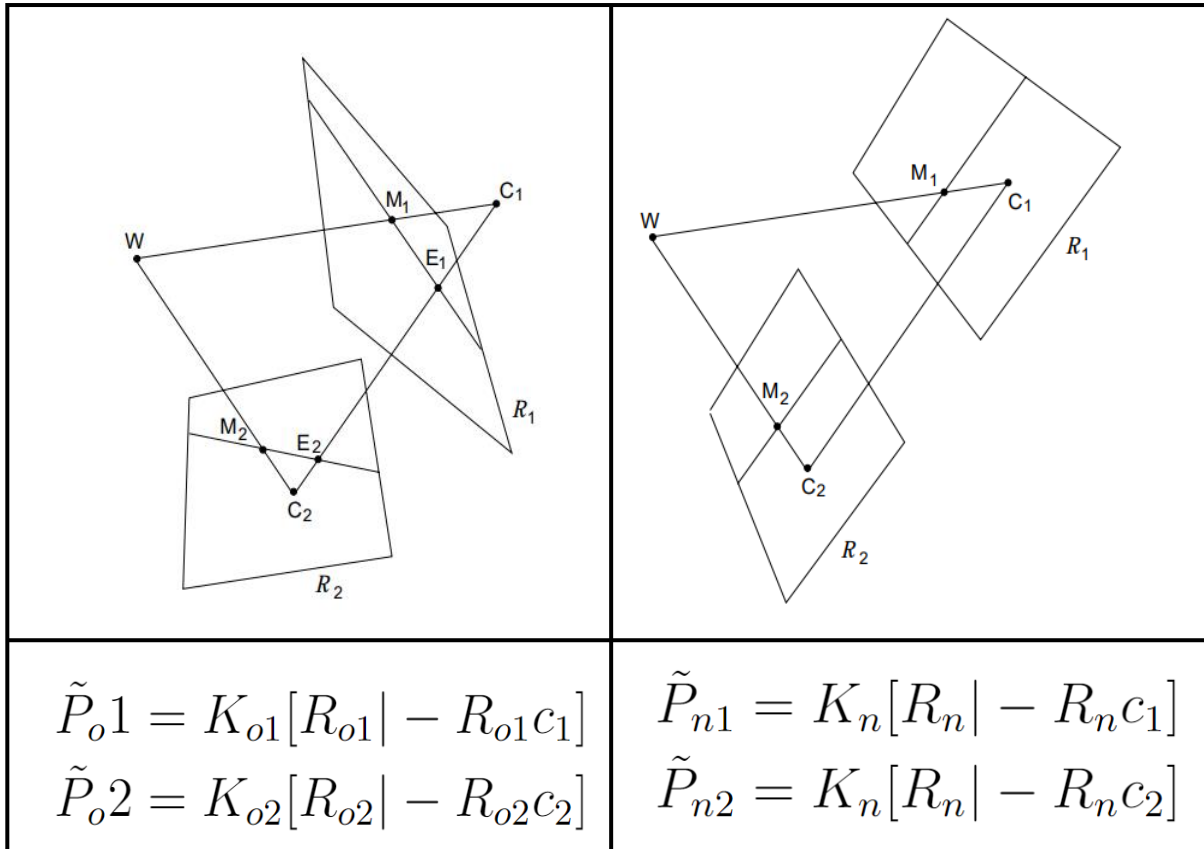


$A_n$

$$A_n = (A_1 + A_2)/2$$

By averaging the two intrinsic parameter

# Rectification



$$w = c_1 + \lambda_{o1} (K_{o1} R_{o1})^{-1} \tilde{m}_{o1}$$

$$w = c_1 + \lambda_{n1} (K_n R_n)^{-1} \tilde{m}_{n1}$$

The world coordinate is same

$$\tilde{m}_{n1} = \lambda_1' (K_n R_n) (K_{o1} R_{o1})^{-1} \tilde{m}_{o1}$$

↓

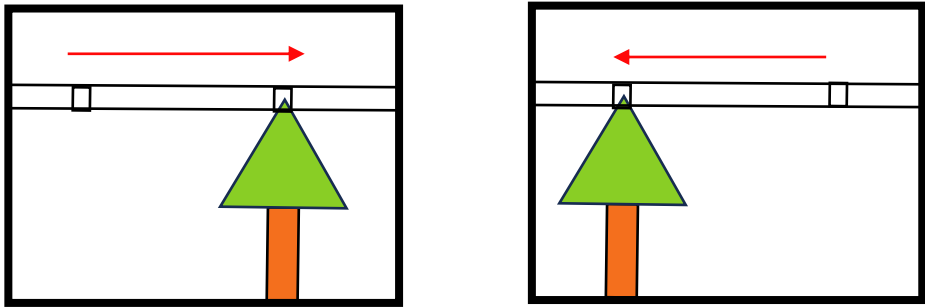
$$H_1 = (K_n R_n) (K_{o1} R_{o1})^{-1}$$

$H_2$  is also calculated in the same way

# Algorithm design

**Rectification**       $\longrightarrow$       We can assume ideal modeling in designing algorithm

- All Epiline is always parallel to the horizontal axis
- All pair of correspondence points share the same vertical axis coordinates



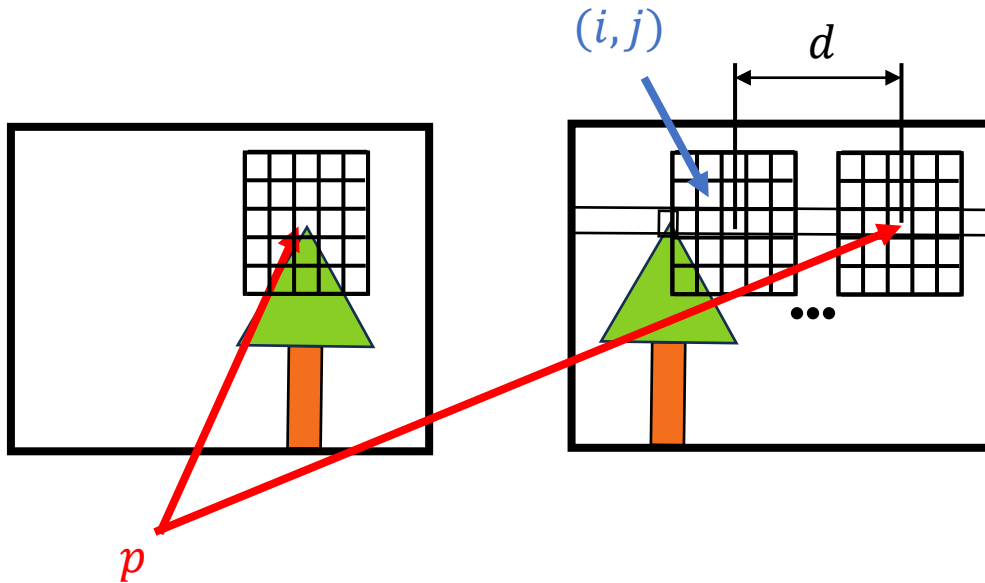
- Starting from own coordinates and moving along the horizontal axis (Direction is different)
- Calculating the disparity with the corresponding point

$$Depth = \frac{f \times B}{Disparity}$$

$\longrightarrow$       **Disparity Map**       $\longrightarrow$       **Depth Map**

# Block Matching

Comparing pixels on a block-by-block basis to find matching points



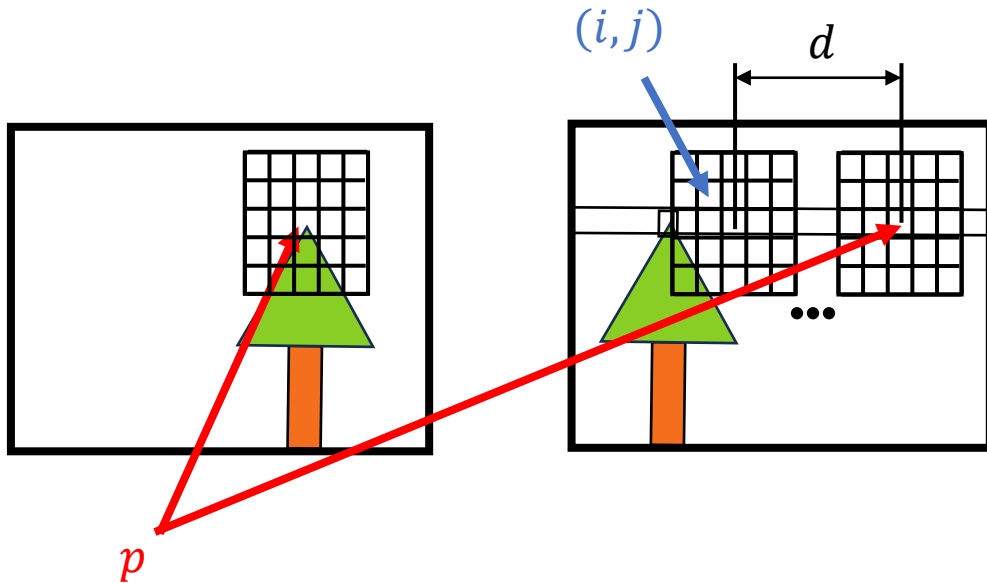
- Comparing pixels one by one has very low accuracy
- It measures how well the features within the block match each other

→ **Cost function**

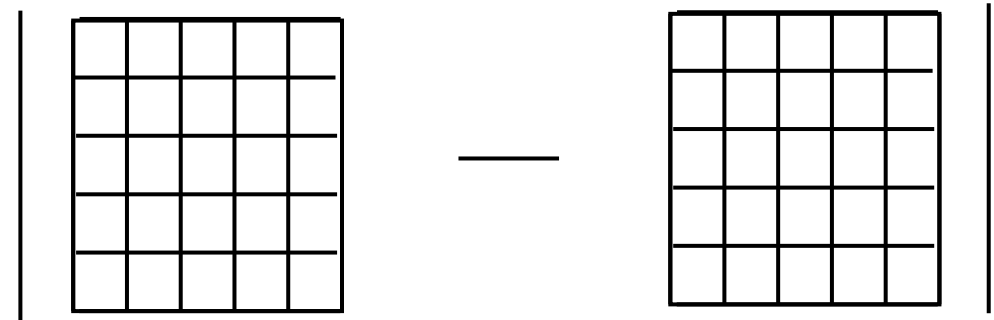
$$C(p, d) = \sum_{(i,j) \in W} f(i, j, p, d)$$

# Matching Cost

## Sum of Absolute Differences



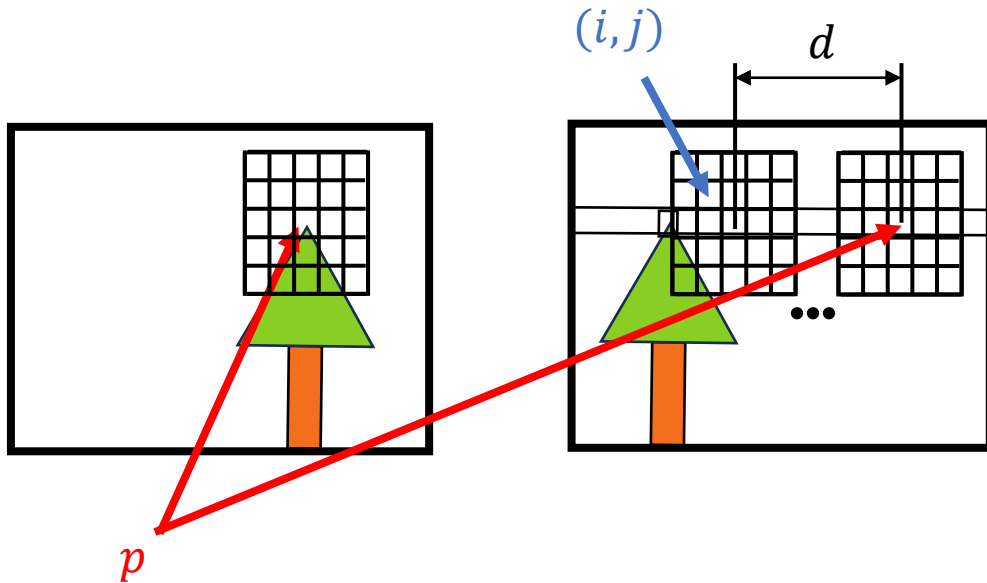
$$C(p, d) = \sum_{(i,j) \in W} |I_l(i, j) - I_r(i - d, j)|$$



- SAD is sensitive to noise and brightness change
- Feature matching is good when cost is low

# Matching Cost

## Sum of Squared Differences



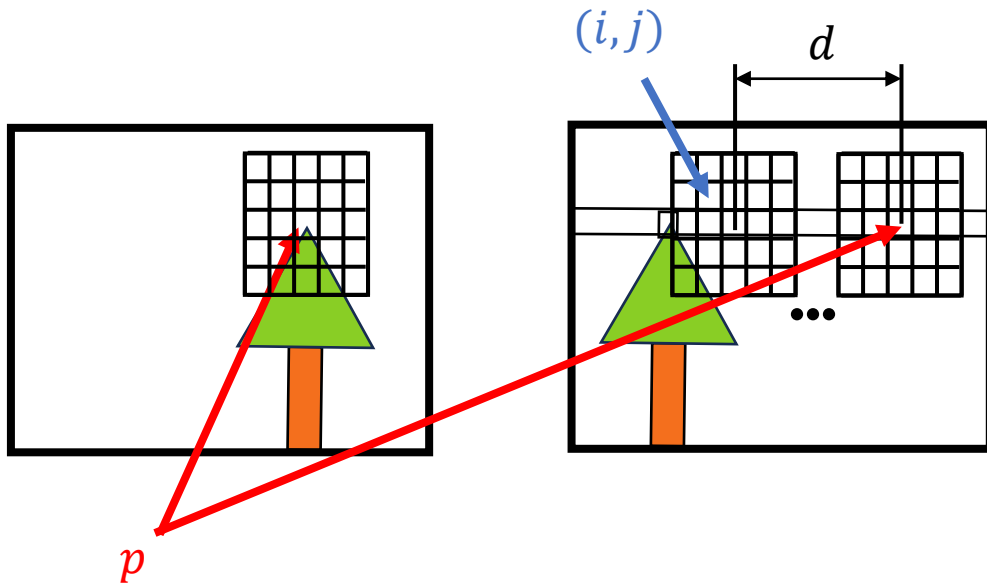
$$C(p, d) = \sum_{(i,j) \in W} |I_l(i, j) - I_r(i - d, j)|^2$$

$$\left| \begin{array}{|c|} \hline \begin{array}{c} \begin{array}{|c|c|c|c|c|} \hline & & & & \\ \hline & & & & \\ \hline & & & & \\ \hline & & & & \\ \hline & & & & \\ \hline \end{array} \\ \hline \end{array} - \begin{array}{|c|} \hline \begin{array}{c} \begin{array}{|c|c|c|c|c|} \hline & & & & \\ \hline & & & & \\ \hline & & & & \\ \hline & & & & \\ \hline & & & & \\ \hline \end{array} \\ \hline \end{array} \right|^2$$

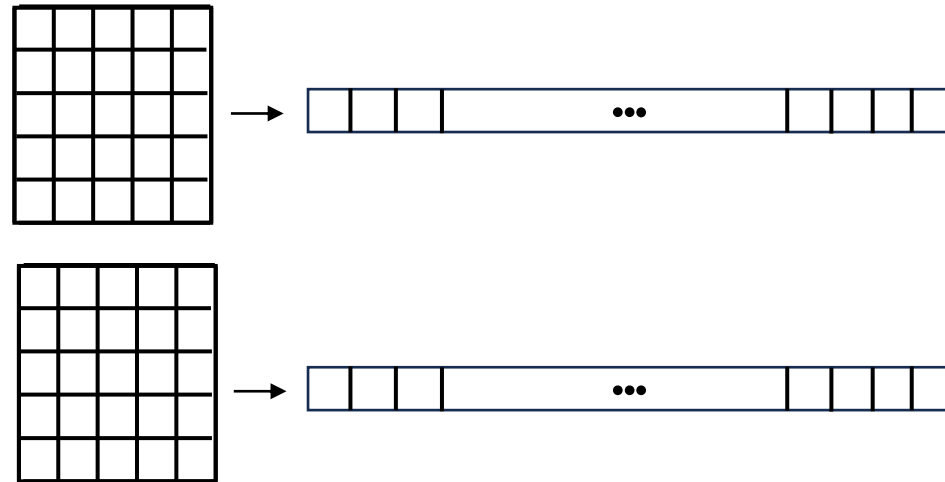
- SAS is sensitive to noise and brightness change
- Feature matching is good when cost is low

# Matching Cost

## Normalized Cross Correlation



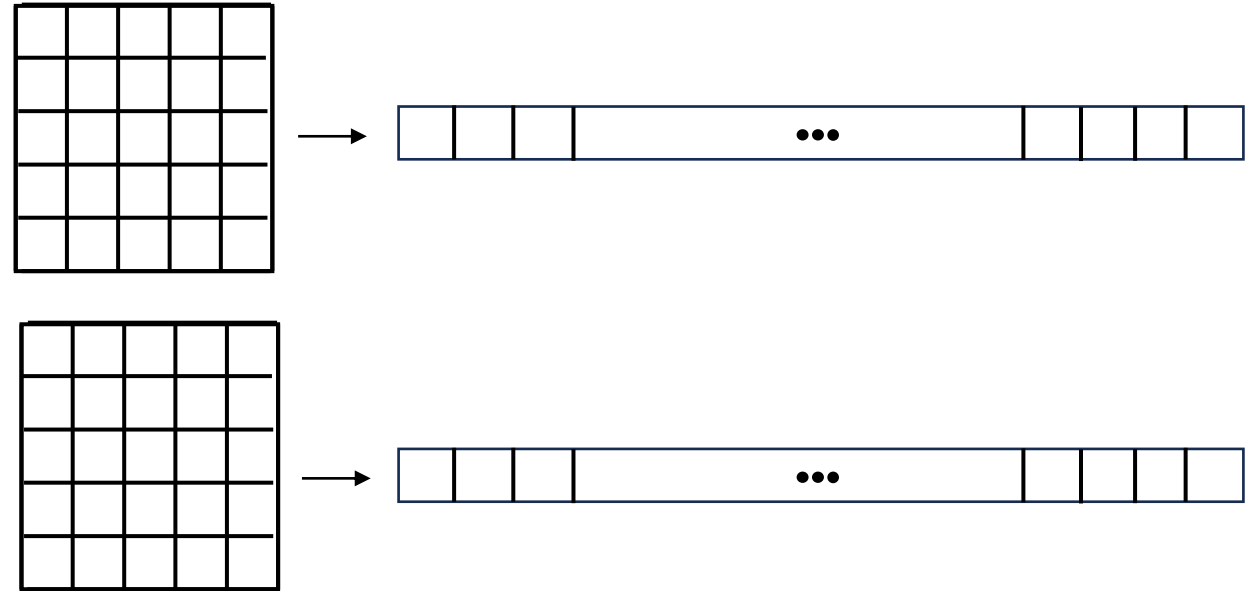
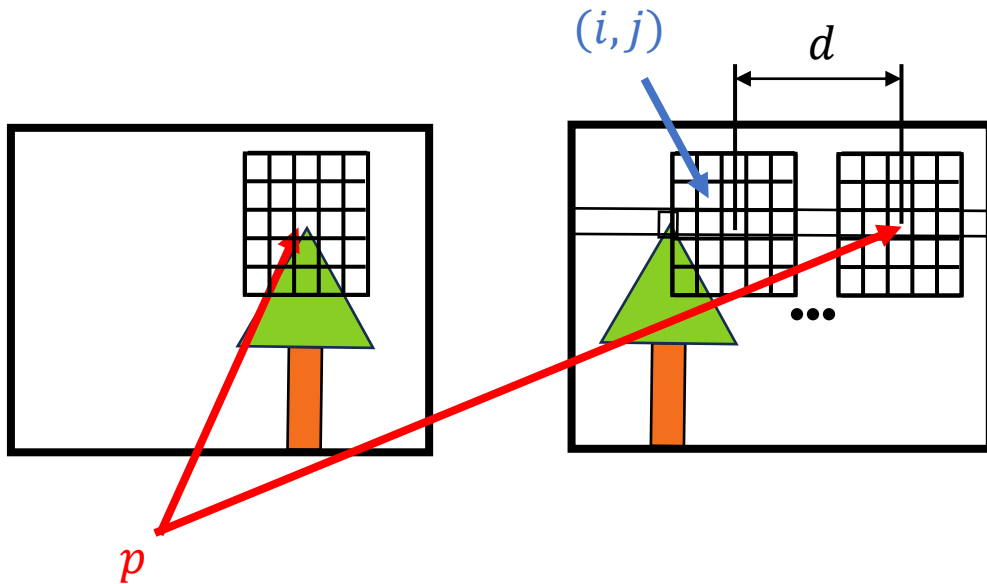
$$C(p, d) = \frac{\sum_{(i,j) \in W} I_l(i, j) I_r(i - d, j)}{\sqrt{\sum_{(i,j) \in W} I_l^2(i, j) \sum_{(i,j) \in W} I_r^2(i - d, j)}}$$



- NCC is less sensitive to noise and brightness change
- Feature matching is good when cost is closer to 1

# Matching Cost

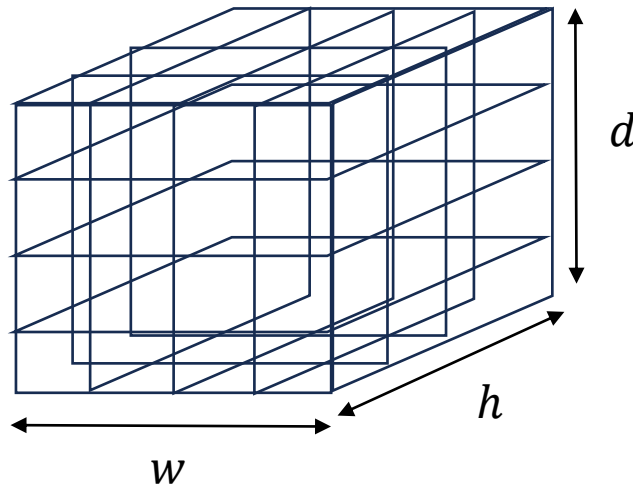
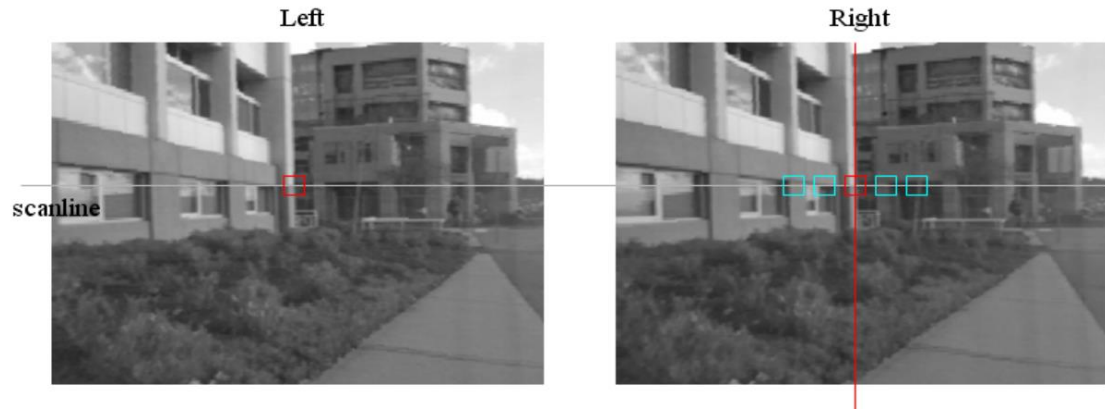
## Census transform



- If pixel values are higher than the central pixel value, assign 1
- If two arrays are different, assign 1
- Cost : How many different value in two array



# Local Matching



## Cost function

$$C(p, d) = \sum_{(i,j) \in W} f(i, j, p, d)$$

→ 3 variable function

## Local Matching

1. Cost calculation
2. Making cost volume
2. Select  $d$  value at each pixel

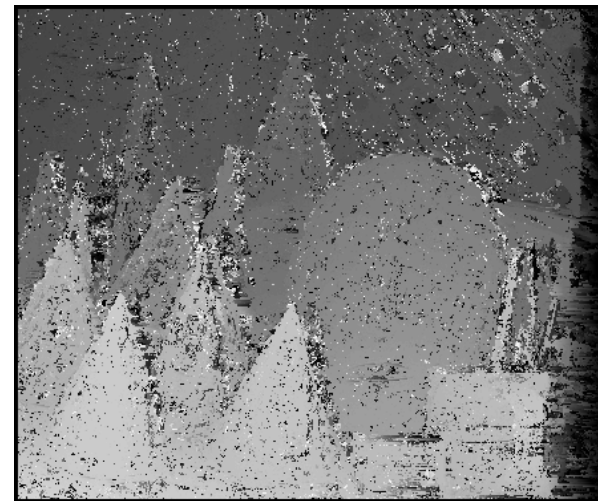
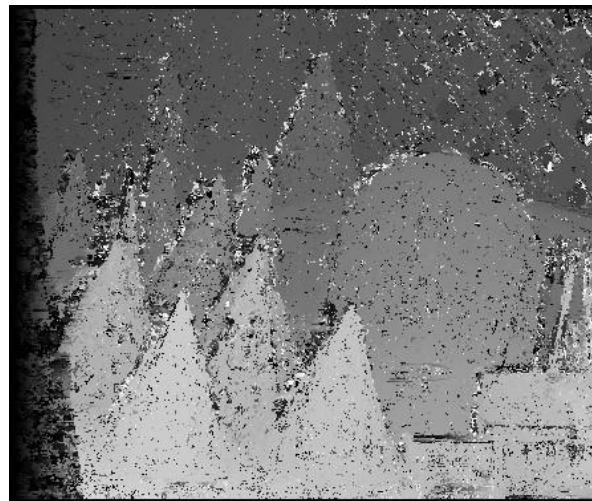
→ The process of finding the corresponding point with the highest similarity at each pixel

# Local matching



## Problem

Continuity of cost volume  
is not considered



# Local matching

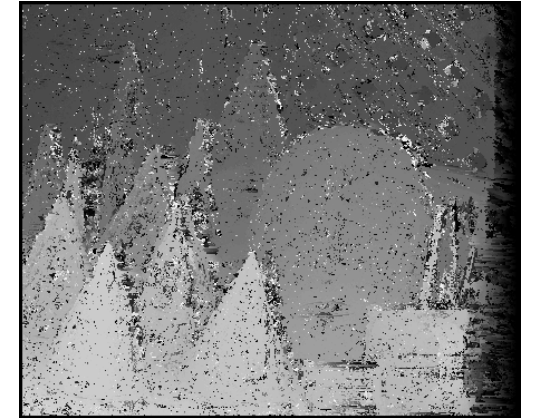
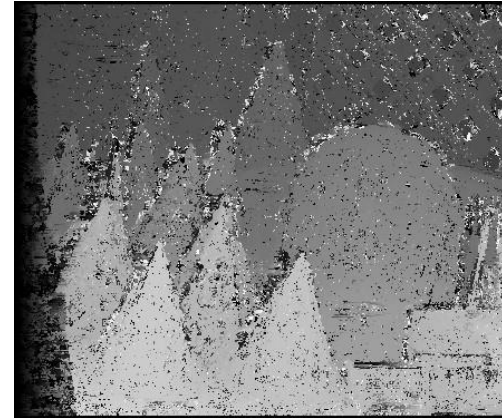
## pros

- Time complexity of algorithm is low

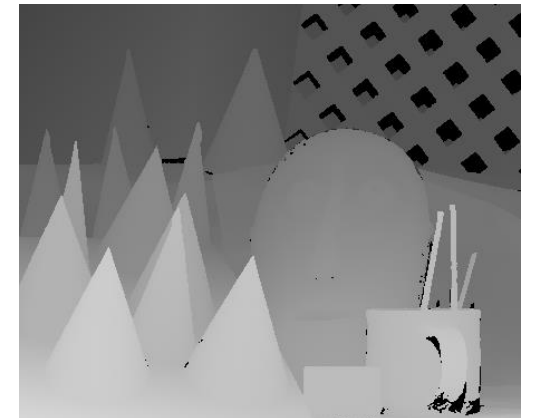
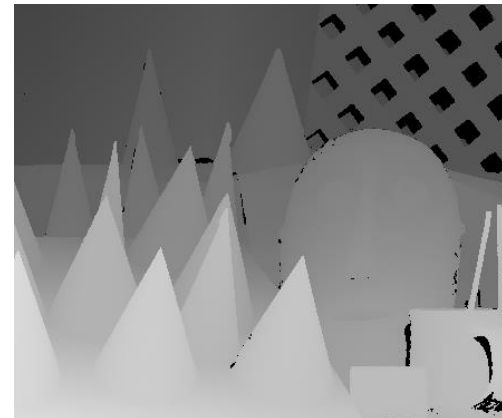
## cons

- It is heavily influenced by noise
- Impossible to make accurate depth map
- Depth map is not continuous

## Local matching



## Ground truth

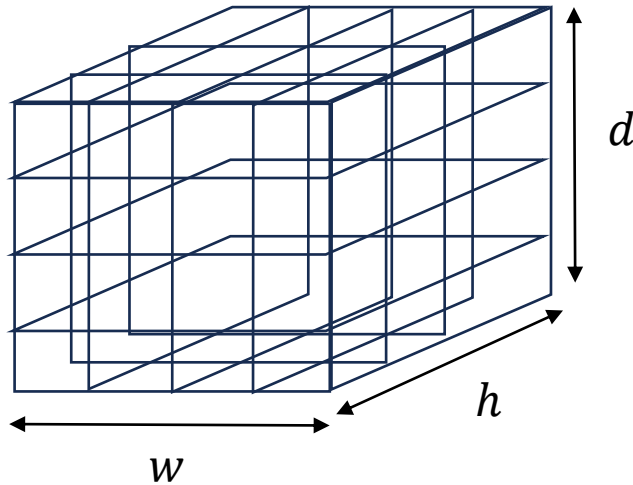


# Energy Function





# Global Matching



1. Cost calculation
2. Making cost volume
3. Defining Energy function
4. Optimize Energy function

Same with  
Local matching



That is, Global matching considers the continuity of cost volume

# Semi Global Matching

$$\mathcal{C} = \mathcal{C}_{data} + \lambda \mathcal{C}_{discon}$$

$$\mathcal{C}_{data}(p, d) = \sum_{p \in W} f(p, d)$$

similarity

continuity

$$\mathcal{C}(d) = \sum_p \left( \mathcal{C}_{data}(\mathbf{p}, d_{\mathbf{p}}) + \frac{\sum_{\mathbf{q} \in \mathcal{N}_{\mathbf{p}}} P_1 \cdot T[|d_{\mathbf{p}} - d_{\mathbf{q}}| = 1] + \sum_{\mathbf{q} \in \mathcal{N}_{\mathbf{p}}} P_2 \cdot T[|d_{\mathbf{p}} - d_{\mathbf{q}}| > 1]}{\quad} \right)$$

# Semi Global Matching

$$\mathcal{C}(d) = \sum_p \left( \mathcal{C}_{data}(\mathbf{p}, d_{\mathbf{p}}) + \sum_{\mathbf{q} \in \mathcal{N}_{\mathbf{p}}} P_1 \cdot T[|d_{\mathbf{p}} - d_{\mathbf{q}}| = 1] + \sum_{\mathbf{q} \in \mathcal{N}_{\mathbf{p}}} P_2 \cdot T[|d_{\mathbf{p}} - d_{\mathbf{q}}| > 1] \right)$$

$\mathcal{N}_{\mathbf{p}}$  : local neighborhood around pixel  $\mathbf{p}$  in the reference image  $I$

$$T(\text{arg}) = \begin{cases} 1 & (\text{arg} = \textit{true}) \\ 0 & (\text{arg} = \textit{false}) \end{cases}$$

$P_1$       Penalty for small disparity change

$P_2$       Penalty for large disparity change

# Semi Global Matching

$$\mathcal{C}(d) = \sum_p \left( \mathcal{C}_{data}(\mathbf{p}, d_{\mathbf{p}}) + \sum_{\mathbf{q} \in \mathcal{N}_{\mathbf{p}}} P_1 \cdot T[|d_{\mathbf{p}} - d_{\mathbf{q}}| = 1] + \sum_{\mathbf{q} \in \mathcal{N}_{\mathbf{p}}} P_2 \cdot T[|d_{\mathbf{p}} - d_{\mathbf{q}}| > 1] \right)$$

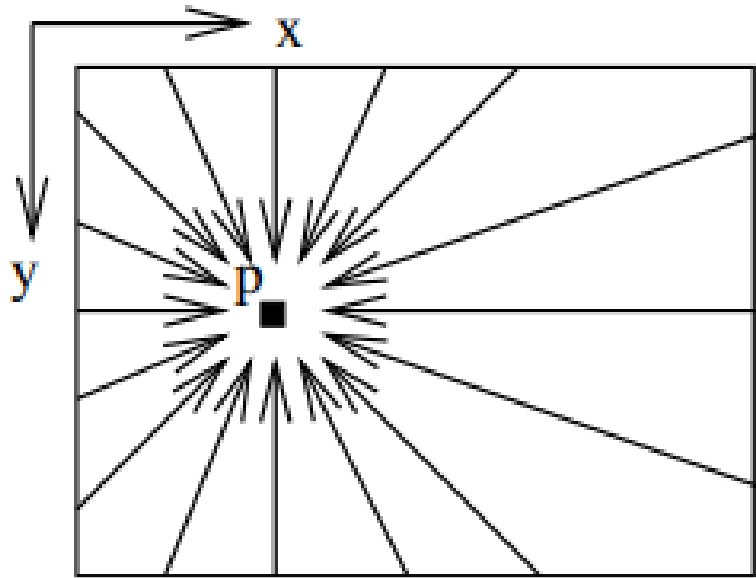
To determine the disparity of the current pixel, it is necessary to simultaneously determine the disparities of adjacent pixels.

2D global optimization(Simultaneously minimizing the disparity value for all image pixels)

➡ N-P complete problem(The time complexity increases exponentially)



# Semi Global Matching



## Assumption

Adjacent pixels coming from different directions do not influence each other

## Result

2D optimization -> Several 1D optimization

# Semi Global Matching

**Energy function**

$$\mathcal{C}(d) = \sum_p \left( \mathcal{C}_{data}(\mathbf{p}, d_{\mathbf{p}}) + \sum_{\mathbf{q} \in \mathcal{N}_{\mathbf{p}}} P_1 \cdot T[|d_{\mathbf{p}} - d_{\mathbf{q}}| = 1] + \sum_{\mathbf{q} \in \mathcal{N}_{\mathbf{p}}} P_2 \cdot T[|d_{\mathbf{p}} - d_{\mathbf{q}}| > 1] \right)$$

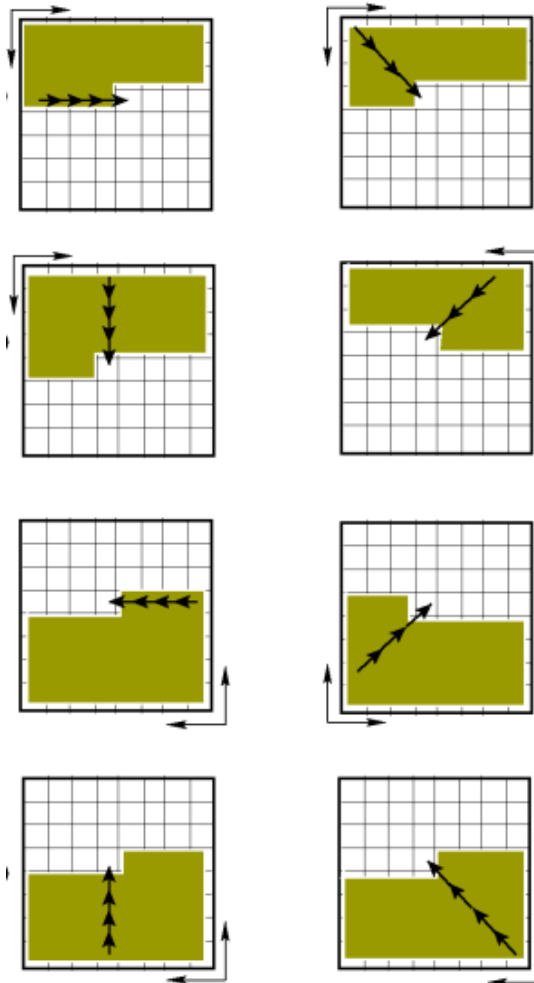
**Path cost**

$$L_r(p, d) = C(p, d) + \min \left\{ \begin{array}{l} L_r(p-r, d), \\ L_r(p-r, d \pm 1) + P_1, \\ \min L_r(p-r, k) + P_2 \end{array} \right\} - \min L_r(p-r, k)$$

**Total cost**

$$C(p, d) = \sum_r L_r(p, d)$$

# Semi Global Matching



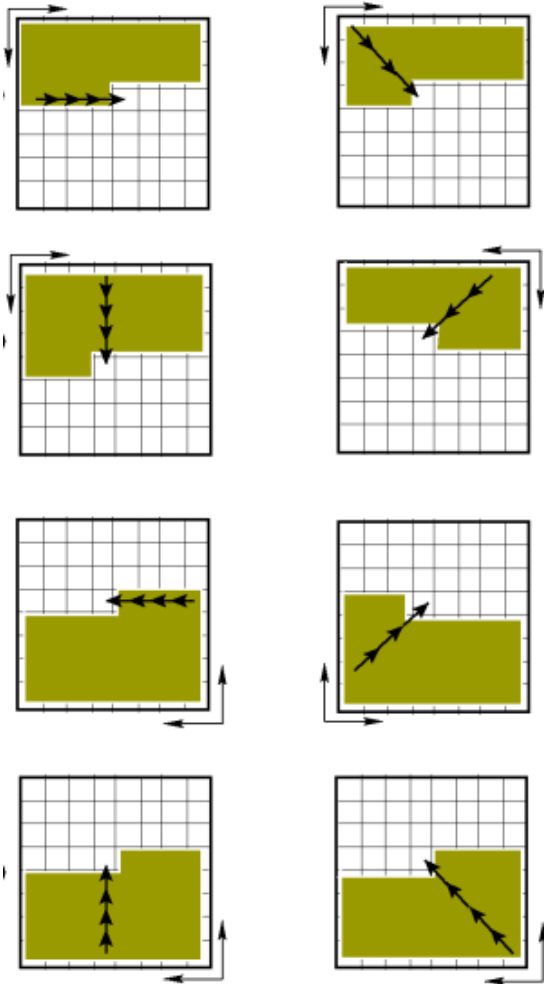
## Dynamic Programming

$$L_r(p, d) = C(p, d) + \min \left\{ \begin{array}{l} L_r(p-r, d), \\ L_r(p-r, d \pm 1) + P_1, \\ \min L_r(p-r, k) + P_2 \end{array} \right\} - \min L_r(p-r, k)$$

$L_r(p, d) \rightarrow$  **4 variable function**

store path cost in  $(width, height, depth, path)$  array

# Semi Global Matching



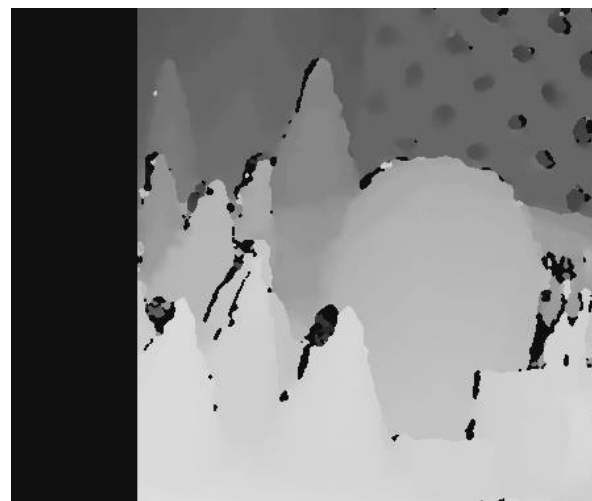
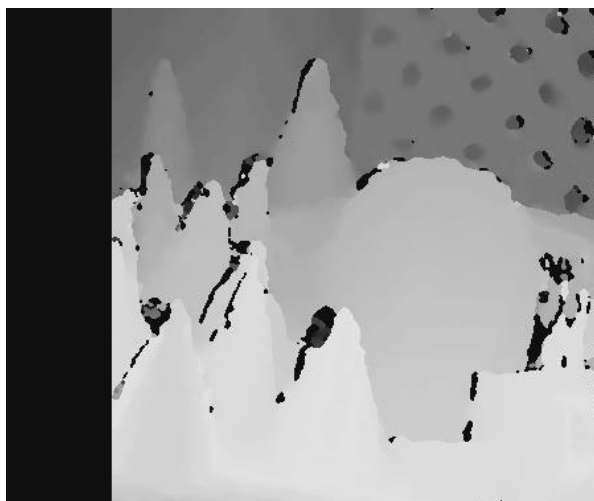
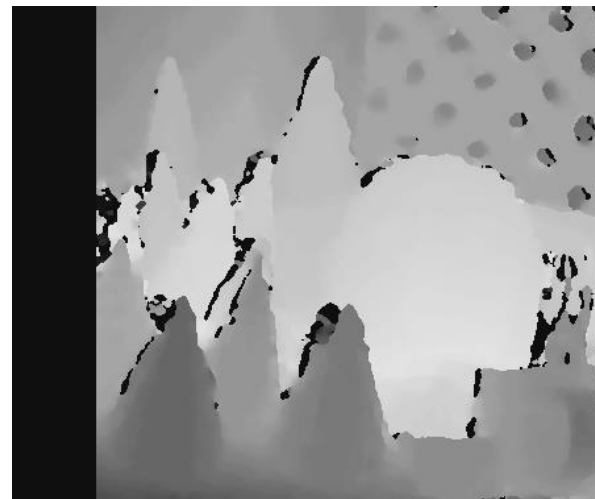
## Parallel processing

SGBM algorithm can perform parallel processing when calculating path costs for each direction



Advantage for real time processing

# Semi Global Matching



Thank You